

Compton scattering of X-rays



Physics

Modern Physics

Production & use of X-rays



Difficulty level

hard



Group size

2



Preparation time

45+ minutes



Execution time

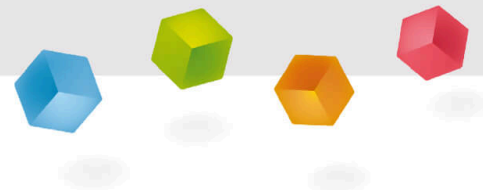
45+ minutes

This content can also be found online at:



<http://localhost:1337/c/5f6090687e9d5b0003e1e80d>

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General information

Application

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Setup

Most applications of X rays are based on their ability to pass through matter. Since this ability is dependent on the density of the matter, imaging of the interior of objects and even people becomes possible. This has wide usage in fields such as medicine or security.

Other information (1/2)

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**Prior****knowledge****Main****principle**

The prior knowledge required for this experiment is found in the Theory section.

During this experiment, the Compton wavelength is determined indirectly with the aid of X-rays. For this purpose, X-rays are scattered on an acrylic glass block. The intensity of the scattered radiation is measured with a counter tube. Then, the Compton wavelength is determined based on the transmission behaviour and on a transmission curve that was measured beforehand.

Other information (2/2)

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**Learning****objective****Tasks**

The goal of this experiment is to get to determine the Compton wavelength of the electron.

1. Determine the transmission of an aluminium absorber as a function of the Bragg angle and plot it as a function of the wavelength of the radiation.
2. Measure the intensity of the radiation that is scattered at an angle of a) 60° b) 90° and c) 120° on an acrylic glass block with and without an absorber.
3. Determine the Compton wavelength of the electron based on the transmission curve.

Theory (1/4)

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The absorption of a material is determined by three different interaction processes. Their relative contributions depend on the atomic number (nuclear charge number) Z and on the mass number A of the shielding material. The most important individual processes are:

- Photoelectric effect; attenuation $\approx Z^4/A$
- Compton scattering; attenuation $\approx Z/A$
- Pair generation; attenuation $\approx Z^2/A$

As a result, the energy-dependent absorption coefficient of a material μ consists of the absorption coefficient of the pair generation μ_{P_α} , of the photoelectric effect μ_{P_h} , and of the Compton effect μ_{C_o} . Two additional mechanisms, the nuclear photoelectric effect and the normal elastic scattering, can usually be neglected for the screening effect.

$$\mu = \mu_{P_\alpha} + \mu_{P_h} + \mu_{C_o}$$

Theory (2/4)

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$$\mu = \mu_{P_\alpha} + \mu_{P_h} + \mu_{C_o}$$

For the X-radiation range that we focus on ($E \approx 1$ -100 keV), μ_{P_α} can be neglected (see Fig. 1). μ_{P_h} is also not relevant for this experiment since only electrons, and not photons, are released. As a result, the detector detects nearly exclusively Compton fractions.

A schematic representation of the Compton effect is shown in Fig. 2. Due to the interaction with a free electron in solid material, the incident photon loses energy and is scattered from its original direction under the scattering angle θ . The previously resting electron absorbs additional kinetic energy and leaves the collision point under the angle ϕ .

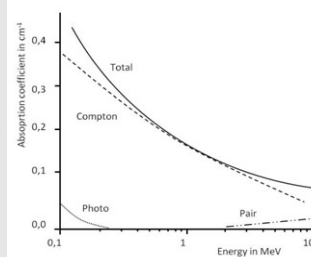


Fig. 1: Absorption coefficient as a function of the energy in the case of aluminium

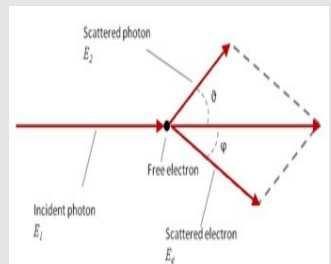


Fig. 2: Momentum and energy relationships in Compton scattering

Theory (3/4)

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Based on the principle of conservation of energy and momentum, the energy of the scattered photon is obtained as a function of the scattering angle:

$$E_2 = \frac{E_1}{1 + \frac{E_1}{m_0 c^2} (1 - \cos(\theta))} \quad (1)$$

After the collision, the photon has a smaller energy E_2 and, therefore, a greater wavelength λ_2 than before the collision. With $E = h\nu$, (1) can be converted into:

$$\frac{1}{h\nu_2} - \frac{1}{h\nu_1} = \frac{1}{m_0 c^2} (1 - \cos(\theta)) \quad (2)$$

Photon energy before or after the collision:
 E_1 or E_2

Planck's constant $h = 6.6256 \cdot 10^{-34} \text{ Js}$

Velocity of light $c = 2.9979 \cdot 10^8 \frac{\text{m}}{\text{s}}$

Restmass of the electron $m_0 = 9.109 \cdot 10^{-31} \text{ kg}$

Scattering angle θ

Photon frequency ν

Theory (4/4)

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With $\lambda = c/\nu$, equation (2) leads to:

$$\lambda_2 - \lambda_1 = \Delta\lambda = \frac{h}{m_0 c} (1 - \cos(\theta)) \quad (3)$$

For 90°-scattering, the wavelength difference, which consists only of the three universal components, leads to the so-called Compton wavelength λ_C for electrons.

$$\lambda_C = \frac{h}{m_0 c} = 2.426 \text{ pm}$$

For the special cases of backward scatter ($\theta = 180^\circ$), the change in wavelength is $\Delta\lambda = 2\lambda_C$.

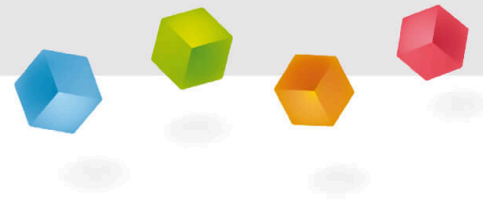
μ=μPα+μPh+μCo" id="MathJax-Element-9-Frame" role="presentati

Equipment

| Position | Material | Item No. | Quantity |
|----------|---|----------|----------|
| 1 | XR 4.0 expert unit, 35 kV | 09057-99 | 1 |
| 2 | XR 4.0 X-ray goniometer | 09057-10 | 1 |
| 3 | XR4 X-ray Plug-in Cu tube | 09057-51 | 1 |
| 4 | XRC 4.0 X-ray characteristics upgrade set | 09135-88 | 1 |

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Setup and Procedure

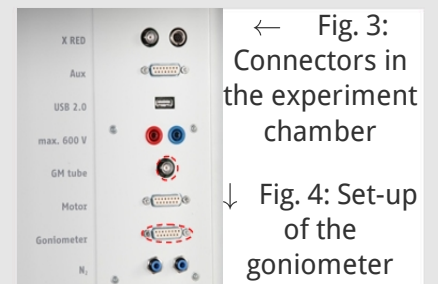


Setup

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Connect the goniometer and the Geiger-Müller counter tube to their respective sockets in the experiment chamber (see the red markings in Fig. 3). The goniometer block with the analyser crystal should be located at the end position on the right-hand side. Fasten the Geiger-Müller counter tube with its holder to the back stop of the guide rails. Do not forget to install the diaphragm in front of the counter tube (see Fig. 4). Insert a diaphragm tube with a diameter of 2 mm into the beam outlet of the tube plug-in unit.

For calibration: Make sure, that the correct crystal is entered in the goniometer parameters. Then, select "Menu", "Goniometer", "Autocalibration". The device now determines the optimal positions of the crystal and the goniometer to each other and then the positions of the peaks.



Procedure (1/3)

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- Connect the X-ray unit via the USB cable to the USB port of your computer (the correct port of the X-ray unit is marked in Figure 5).
- Start the “measure” program. A virtual X-ray unit will be displayed on the screen.
- You can control the X-ray unit by clicking the various features on and under the virtual X-ray unit. Alternatively, you can also change the parameters at the real X-ray unit. The program will automatically adopt the settings.



Fig. 5: Connection of the computer

Procedure (2/3)

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Fig. 6: Part of the user interface of the software

- If you click the experiment chamber (see the red marking in Figure 6), you can change the parameters of the experiments. Select the parameters for task 1 as shown in Figure 7.
- If you click the X-ray tube (see the red marking in Figure 6), you can change the voltage and current of the X-ray tube. Select the parameters as shown in Fig. 8.

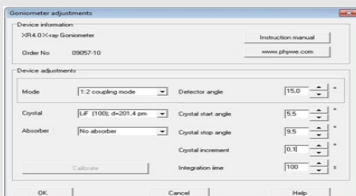


Fig. 7: Settings of the goniometer, task 1

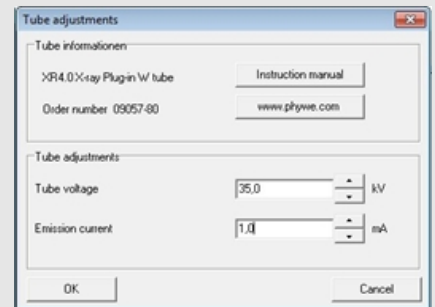
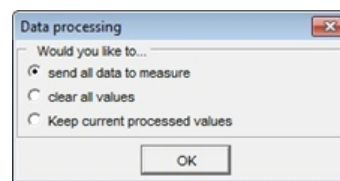
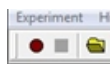


Fig 8: Voltage and current settings

Procedure (3/3)

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- Start the measurement by clicking the red circle:
- After the measurement, the following window appears:



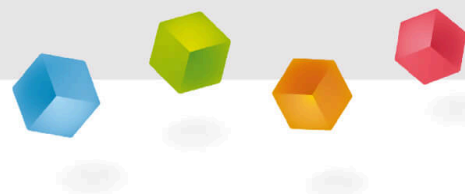
- Select the first item and confirm by clicking OK. The measured values will now be transferred directly to the "measure" software.

Overview of the goniometer and X-ray unit settings:

- 1:2 coupling mode
- Gate time 100 s (gate timer); angle step width 0.1°
- Scanning range $5.5^\circ < \theta < 9.5^\circ$
- Anode voltage $U_A = 35 \text{ kV}$; anode current $I_A = 1 \text{ mA}$

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Evaluation



Task 1

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Task 1: Determine the transmission of an aluminium absorber as a function of the Bragg angle and plot it as a function of the wavelength of the radiation.

Based on the glancing angles θ as well as on the Bragg relationship, the associated wavelengths λ are obtained:

$$2d \sin(\theta) = n\lambda \quad (4)$$

with $d = 201.4 \text{ pm} = \text{LiF-(200)}$ interplanar spacing and here: $n = 1$.

For a given gate time Δt and the pulse rate n the total number of incidents N is $N = n \cdot \Delta t$.
For the measured number of incidents N , the relative error of N is given by the ratio:

$$\frac{\Delta N}{N} = \frac{1}{\sqrt{N}} \quad (5)$$

Task 1 (part 2)

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At high pulse rates, the dead time τ of the counter tube must also be taken into consideration, since it does not register all of the incident photons. For the GM counter tube that is used in this experiment, it is $90 \mu\text{s}$. The true pulse rate n^* can be obtained from the measured pulse rate n with the aid of:

$$n^* = \frac{n}{1 - n\tau} \quad (6)$$

Correct the measured count rate in an angle range of $5.5^\circ < \theta < 9.5^\circ$ and with the dead time $\tau = 90 \mu\text{s}$ of the Geiger-Müller counter tube. This can be done using the software measure:

$$T(\lambda) = \frac{n_2^*(\text{with absorber})}{n_1^*(\text{without absorber})}$$

It is then plotted as a function of λ (see Fig. 9)

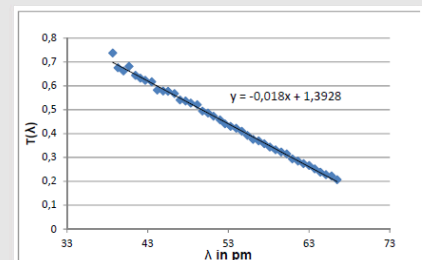


Fig. 9: Transmission curve of aluminium in a narrow wavelength range

Task 2 and 3

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Task 2 and 3: Measure the intensity of the radiation that is scattered at an angle of 60°, 90° and 120° on an acrylic glass block with and without an absorber and determine the Compton wavelength of the electron based on the transmission curve.

In this experiment, the aluminium absorber acts as a kind of strong colour filter. It absorbs shorter wavelengths less strongly than longer wavelengths. This means that if it is positioned in front of the scatterer, it has a different effect than in the long-wave scattered radiation behind the scatterer. This enables the determination of the wavelength of the scattered and non-scattered radiation. By placing the absorber in the ray path between the X-ray tube and the scatterer (position 1, Fig. 8), you can determine the transmission $T_1 = n_4/n_3$ of the still non-scattered X-radiation. When the absorber is in position 2, you obtain the transmission of the scattered X-radiation. Since we have determined the dependence of the transmission of aluminium on the wavelength in task 1, we can now directly infer from the transmission to the wavelength of the X-radiation that passes through the absorber. The two different transmission coefficients that are obtained from the 90° scattering then lead to the corresponding wavelengths.

Sample results

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Fig. 9 shows transmission curve of aluminium in a narrow wavelength range including the equation of the regression line:

$$y = -0.018x + 1.3928$$

The results from Task 2 for n_3^* , n_4^* and n_5^* are listed in table 1

Sample calculation for the 90° scattering:

$$T_1 = \frac{n_4^* \cdot \Delta t}{n_3^* \cdot \Delta t} = 0.347 \pm 1.27\%$$

$$T_2 = \frac{n_5^* \cdot \Delta t}{n_3^* \cdot \Delta t} = 0.315 \pm 1.32\%$$

The deviation of $\pm 1.27\%$ and $\pm 1.32\%$ is calculated with the aid of equation (2) and the following equation (use the total number of incidents N^* for the calculation):

$$\Delta T_1 = \sqrt{\frac{1}{|\Delta N_4^*|} + \frac{1}{|\Delta N_3^*|}}$$

Table 1 sample results

| θ | n_3^* | n_4^* | n_5^* | T_1 | T_2 | λ_1 | λ_2 | $\Delta\lambda$ |
|----------|---------|---------|---------|-------|-------|-------------|-------------|-----------------|
| 60° | 241.121 | 86.625 | 76.017 | 0.347 | 0.315 | 58.095 | 59.871 | 1.78 |
| 90° | 178.833 | 59.315 | 51.235 | 0.332 | 0.286 | 58.936 | 61.482 | 56 |
| 120° | 216.124 | 70.444 | 58.81 | 0.336 | 0.272 | 58.716 | 62.263 | 56 |

Sample results (part 2)

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The relative error only takes into account the statistical errors. Systematic errors (see note) are not considered.

Based on the linear equation of the regression line in Fig. 9, the associated wavelengths for the 90° scattering result as 58.93 pm and 61.48 pm. This results in a wavelength difference of $\Delta\lambda = \lambda_C = 2.56$ pm, which is very close to the theoretical value of $\lambda_C = 2.426$ pm.

The experiment show that with decreasing scattering angle, the difference in wavelength also decreases.

Note

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There is a systematic error, since

- X-rays are also diffracted at the aluminium sheet so that they might actually enter the counter directly.
- The incident x-radiation is polychromatic
- The geometry of the scattering region is not spherical

The error caused by fluorescence is negligible because the corresponding radiation energy is too low to be registered.