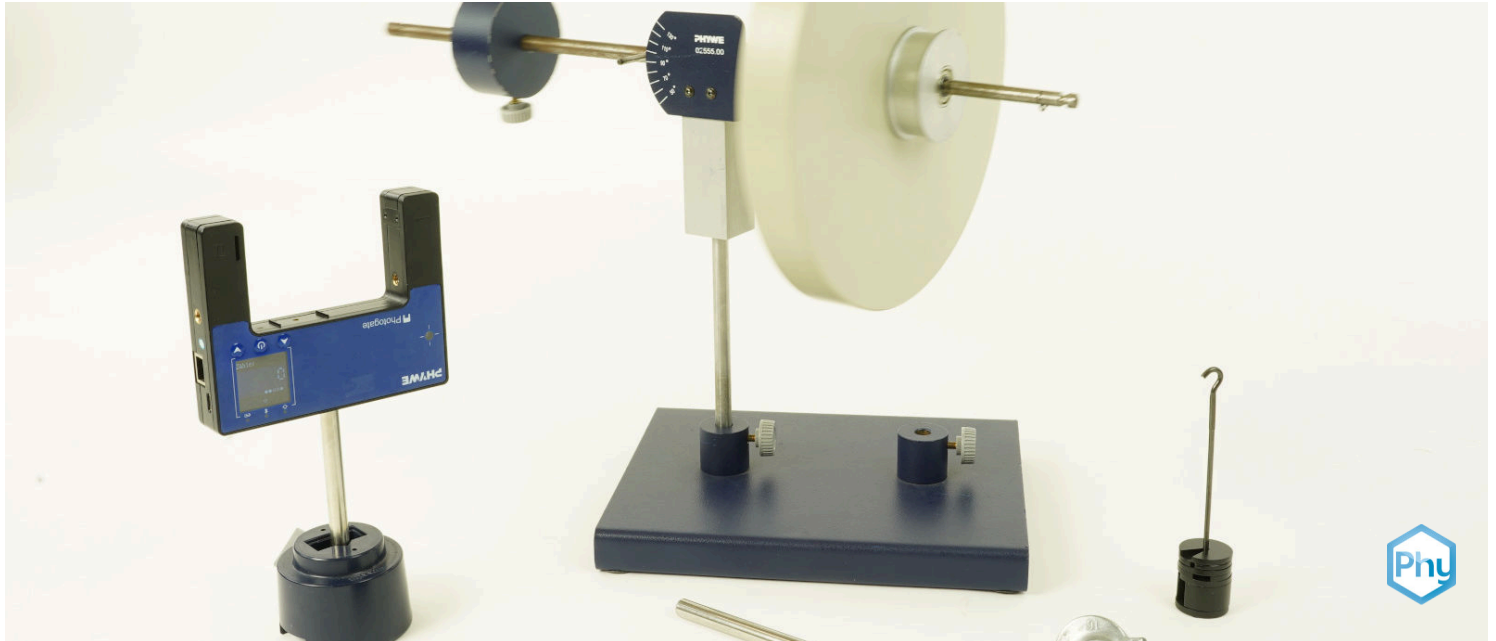


# Laws of gyroscopes/ 3-axis gyroscope



Physics

Mechanics

Circular motion &amp; rotation



Difficulty level

hard



Group size

-



Preparation time

45+ minutes



Execution time

45+ minutes

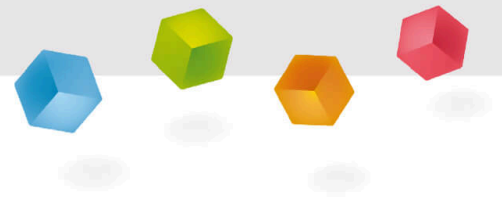
This content can also be found online at:



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# General information



## Application



Fig.1: Experimental set-up

The moment of inertia and the angular acceleration are fundamental for the field of mechanics. As such it's understanding is very important for the study of this field. A gyroscope is effective tool to accomplish this.

## Other information (1/2)

The prior knowledge for this experiment is found in the Theory section.

### Prior knowledge

### Main principle

The momentum of inertia of the gyroscope is investigated by measuring the angular acceleration caused by torques of different known values. In this experiment, two of the axes of the gyroscope are fixed. The relationship between the precession frequency and the gyro-frequency of the gyroscope with 3 free axes is examined for torques of different values applied to the axis of rotation. If the axis of rotation of the force-free gyroscope is slightly displaced, a nutation is induced. The nutation frequency will be investigated as a function of gyro-frequency.

## Other information (2/2)

The goal of this experiment is to investigate the moment of inertia.

### Learning objective

1. Determination of the momentum of inertia of the gyroscope by measurement of the angular acceleration.
2. Determination of the momentum of inertia by measurement of the gyro-frequency and precession frequency. Investigation of the relationship between precession and gyrofrequency and its dependence on torque.

### Tasks

3. Investigation of the relationship between nutation frequency and gyro-frequency.

## Theory (1/5)

### Determination of the momentum of inertia of the gyroscope disk

If the gyroscope disk is set to rotate by means of a falling mass  $m$  (Fig. 2), the following relation is valid for the angular acceleration:

$$\frac{d\omega_R}{dt} = \alpha = \frac{M}{I_P} \quad (1)$$

( $\omega_R$  = angular velocity;  $\alpha$  = angular acceleration;  $I_P$  = polar momentum of inertia;  $M = F \cdot r$  = torque)

Fig. 2: Schematic representation of the experimental set-up to determine the momentum of inertia of the gyroscope disk

## Theory (2/5)

According to the law of action and reaction, the force which causes the torque is given by the following relation:

$$F = m \cdot (g - a) \quad (2)$$

( $g$  terrestrial gravitational acceleration;  $a$  = trajectory acceleration)

The following relations are true for the trajectory acceleration  $a$  and the angular acceleration  $\alpha$ :

$$a = \frac{2h}{t_F^2}; \alpha = \frac{a}{r} \quad (3)$$

( $h$  = dropping height of the accelerating mass,  $t_F$  = falling time;  $r$  = radius of the thread drum).  
Introducing (2) and (3) into (1), one obtains:

$$t_F^2 = \frac{2I_P + 2mr^2}{mgr^2} \cdot h \quad (4)$$

In general, the following is valid for the momentum of inertia of a disk:

$$I_P = \frac{1}{2}MR^2 = \frac{\pi}{2} \cdot R^4 \cdot d \cdot \rho \quad (5)$$

## Theory (3/5)

### Determination of the precession frequency

Let the symmetrical gyroscope G in Fig. 3, which is suspended so as to be able to rotate around the 3 main axes, be in equilibrium in horizontal position with counterweight C. If the gyroscope is set to rotate around the x-axis, with an angular velocity  $\omega$  the following is valid for the angular momentum L, which is constant in space and in time:

$$L = I_P \cdot \omega_R \quad (6)$$

Adding a supplementary mass  $m^*$  at the distance  $r^*$  from the support point induces a supplementary torque  $M^*$ , which is equal to the variation in time of the angular momentum and parallel to it.

Fig. 3: Schematic representation of the gyroscope submitted to forces

## Theory (4/5)

$$M^* = m^* \cdot g r^* = \frac{dL}{dt} \quad (7)$$

Due to the influence of the supplementary torque (which acts perpendicularly in this particular case), after a lapse of time  $dt$ , the angular momentum L will rotate by an angle  $d\phi$  from its initial position (Fig. 4).

$$dL = L d\phi \quad (8)$$

The gyroscope does not topple under the influence of the supplementary torque, but reacts perpendicularly to the force generated by this torque. The gyroscope, which now is submitted to gravitation, describes a so-called precession movement.

The angular velocity  $\varphi_P$  of the precession fulfills the relation:

$$\omega_P = \frac{d\phi}{dt} = \frac{dL}{L dt} = \frac{dL}{I_P \cdot \omega_R dt} = \frac{m^* \cdot g r^*}{I_P \cdot \omega_R} \quad (9)$$

## Theory (5/5)

Taking  $\omega_P = 2\pi/t_P$  and  $\omega_R = 2\pi/t_R$  one obtains:

$$\frac{1}{t_r} = \frac{m^* \cdot g r^*}{4\pi^2} \frac{1}{I_P} \cdot t_P \quad (10)$$

Fig. 4: Precession of the horizontal axis of the gyroscope

## Equipment

Position	Material	Item No.	Quantity
1	Gyroscope with 3 axes	02555-00	1
2	Additional gyro-disk w. c-weight	02556-00	1
3	Digital stopwatch, 24 h, 1/100 s and 1 s	24025-00	1
4	Weight holder, 10 g	02204-01	1
5	Slotted weight, silver bronze, 10 g	02205-03	4
6	Slotted weight, silver bronze, 50 g	02206-03	1
7	Cobra SMARTsense Dual Photogate - Double light barrier 0 ... $\infty$ s (Bluetooth + USB)	12945-00	1
8	Fishing line, l. 5m	02089-01	1
9	Barrel base expert	02004-00	1
10	Measuring tape, l = 2 m	09936-00	1
11	Support rod, stainless steel, l = 250 mm, d = 10 mm	02031-00	1
12	Right angle boss-head clamp	37697-00	1

# Setup and Procedure

## Setup and Procedure (1/2)

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The experimental set-up is shown in Fig 1:

- To start with, the polar momentum of inertia  $I_p$  of the gyroscope disk must be determined. For this, the gyroscope is fixed with its axis directed horizontally and positioned on the experimenting table in such a way that the thread drum projects over the edge of the table (Fig. 2). The thread is wound around the drum and the accelerating mass  $m$  ( $m = 60$  g; plate with 5 slotted weights) is fastened to the free end of the thread. Several experiments are carried out for different drop heights  $h$  of the accelerating mass, from which the corresponding average falling time  $t_F$  from the moment the gyroscope disk is released until the mass touches the floor is determined. The diagram of  $t_F^2$  versus  $h$  is plotted and the moment of inertia of the gyroscope disk is determined from the slope of the straight line (Fig. 5).

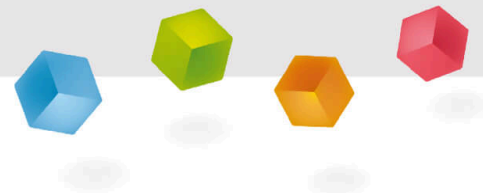


## Setup and Procedure (2/2)

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- The gyroscope, on which no forces act, and which can move freely around its 3 axes, is wound up and the duration  $t_R$  of one revolution (rotation frequency) is determined by measure of the light barrier in the measurement mode "running time", with the axis of the gyroscope lying horizontally. Immediately after this, a mass  $m^* = 30\text{ g}$  is hung at a distance  $r^* = 27\text{ cm}$  into the groove at the longer end of the gyroscope axis. The duration of half a precession rotation  $t_P/2$  must now be determined with a manual stopwatch (this value must be multiplied by two for the evaluation). The mass is then removed, so the gyroscope axis can regain immobility, and  $t_R$  can be determined again. The inverse of the average value from both measurements of  $t_R$  is entered into a diagram above precession time  $t_P$ . In the same way, the other measurement points are recorded for decreasing number of gyroscope rotations. The slope of the resulting straight line allows to calculate the momentum of inertia of the gyroscope disk (Fig. 6).
- If a slight lateral blow is given against the axis of the rotating gyroscope on which no forces are acting, the gyroscope starts describing a nutation movement. The duration of one nutation  $t_R$  is determined with the manual stopwatch and this is plotted against the duration of one revolution  $t_R$ , which is again determined by measurement of the light barrier (Fig. 7).

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## Evaluation

## Task 1

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From the slope of the straight line  $t_F^2 = f(h)$  from Fig. 5, one obtains the following value for the momentum of inertia of the gyroscope disk:

$$I_P = (8.83 \pm 0.15) \cdot 10^{-3}$$

Taking the corresponding values for the radius  $R$  and the thickness  $d$  of the circular disk, and the specific weight of plastic  $\rho = 0.9 \text{ g/cm}^3$ , one obtains from (4):

$$I_P = 8.91 \cdot 10^{-3}$$

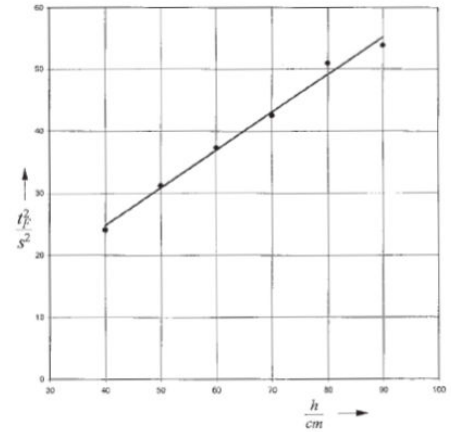


Fig. 5: Determination of the momentum of inertia from the slope of straight line  $t_F^2 = f(h)$ .

## Task 2

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According to (10), Fig. 6 shows the linear relation between the inverse of the duration of a revolution  $t_R$  of the gyroscope disk and the duration of a precession revolution  $t_P$  for two different masses  $m^*$ . The slopes of the straight lines allow calculating the values of the momentum of inertia, for which one obtains:

$I_P = (8.89 \pm 0.15) \text{ kgm}^2$  for  $m^* = 0.03 \text{ kg}$  and  $I_P = (9.29 \pm 0.17) \text{ kgm}^2$  for  $m^* = 0.06 \text{ kg}$ .

The double value of the torque causes the doubling of the precession frequency. If  $m^*$  is hung into the forward groove of the gyroscope axis, or if the direction of rotation of the disk is inverted, the direction of rotation of the precession is also inverted. If the supplementary disk identical to the gyroscope disk is used too, and both are caused to rotate in opposite directions, no precession will occur when a torque is applied.

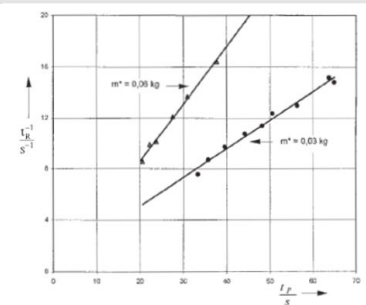


Fig. 6: Determination of the momentum of inertia from the slope of straight line  $1/t_R = f(t_P)$ .

## Task 3

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Fig. 7 represents the relation

$$\omega_N = k\omega_R; t_R = kt_N \quad (11)$$

between the nutation frequency  $\omega_N$  and rotation frequency  $\omega_R$ . The constant  $k$  depends on the different moments of inertia relative to the principal axes of rotation.

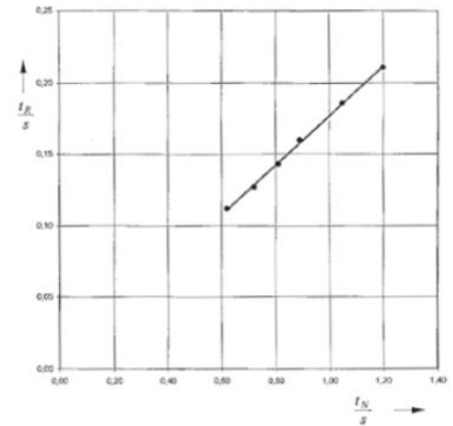


Fig. 7: Nutation time as a function of time for one revolution.