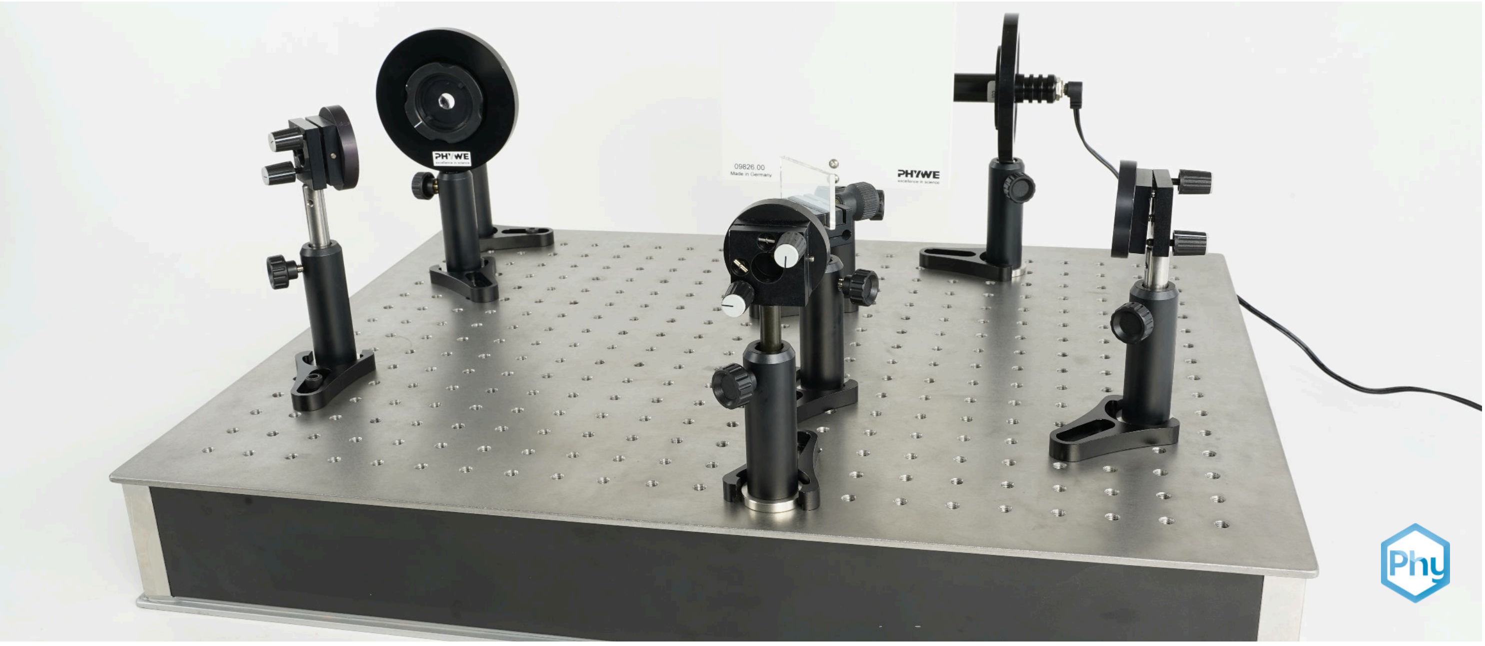
Michelson interferometer - High Resolution



Physics › Light & Optics › Diffraction & interference

Difficulty level	Group size	Preparation time	Execution time
hard	-	45+ minutes	45+ minutes

This content can also be found online at:

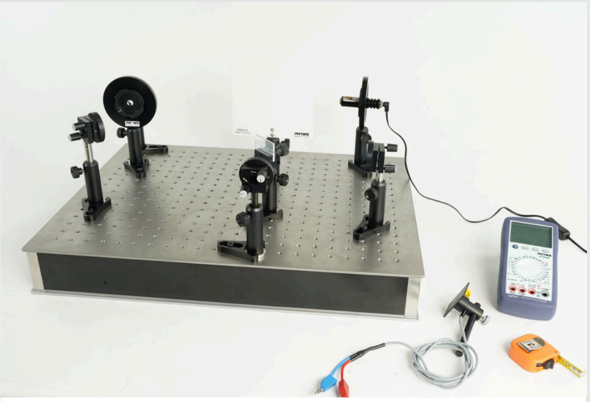
http://localhost:1337/c/66b0c7454e57710002247fda

PHYWE

General information

Application

PHYWE



Setup

The Michelson interferometer (among other interferometer configurations) is employed in many scientific experiments and became well known for its use by Albert Michelson and Edward Morley in the famous Michelson–Morley experiment (1887) in a configuration which would have detected the earth's motion through the supposed luminiferous aether that most physicists at the time believed was the medium in which light waves propagated.

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Other information (1/2)

PHYWE

Prior knowledge

The proir knowlege required for this experiment can be found in the theory section.

Scientific principle

With the aid of two mirrors in a Michelson arrangement, light is brought to interference. While moving one of the mirrors, the alteration in the interference pattern is observed and the wavelength of the laser light determined.

Other information (2/2)

PHYWE

Learning objective

The goal of this experiment is to investigate the principle behind the Michelson interferometer.

Tasks

Construction of a Michelson interferometer using separate components. The interferometer is used to determine the wavelength of the laser light. The contrast function K is qualitatively recorded in order to determine the coherence length with it.

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Theory (1/3)

If two waves of the same frequency ω but of different amplitude and different phase impinge on one point they are superimposed, or interfere, so that:

$$y = a_1 \sin(\omega t - \alpha_1) + \alpha_2 \sin(\omega t - \alpha_2)$$

The resulting wave can be described as

$$y = A \sin(\omega t - \alpha)$$

with the amplitude

$$A^2 = {a_1}^2 + {a_2}^2 + 2a_1 a_2 \cos\delta \qquad (1)$$

and the phase difference $\delta = \alpha_1 - \alpha_2$

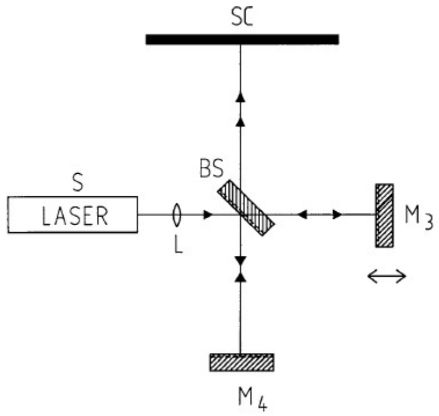


Fig. 1: Michelson arrangement for Interference. S represents the light source; SC the detector (or the position of the screen).

Theory (2/3)

In a Michelson interferometer, light is split up into two beams by a half-silvered glass plate (amplitude splitting), reflected by two mirrors, again brought to interference behind the glass plate (Fig. 1). Since only extensive luminous spots can exhibit circular interference fringes, the light beam is expandedlens L.

If one replaces the real mirror M_3 between the laser and the glass plate by a virtual image M_r', which is formed by reflection at the glass plate, a point P of the real light source is formed as the points P' and P'' of the virtual light sources L_1 and L_2.

Due to the different light paths, using the designations in Fig. 2 the phase difference is given by:

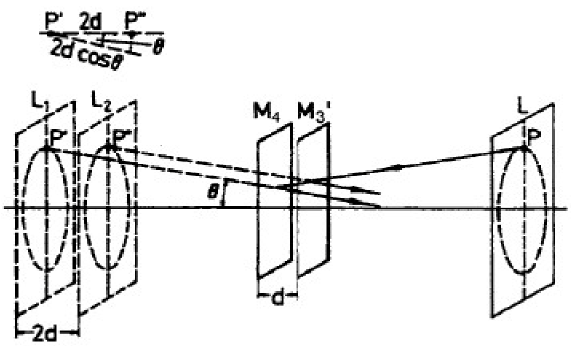


Fig. 2: Formation of circular interference fringes.

Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Theory (3/3)

PHYWE

$\delta = \frac{2\pi}{\lambda} 2d \cos\Theta$ (2)

where λ is the wavelength of the light used in the experiment.

The intensity distribution $a_1 = a_2 = a$ for according to (1) is: $I \sim A^2 = 4a^2 \cos^2 \frac{\delta}{2}$ (3)

Maxima thus occur if δ is a multiple of 2π, i.e. from equation (2), if $2d \cos\Theta = m\lambda$; $m = 1, 2, \ldots$ (4)

i.e. circles are produced for a fixed value of m and d since Θ remains constant (see Fig. 2). If one alters the position of the movable mirror M_3 (cf. Fig.3) such that d, e.g., decreases, according to (4), the circular fringe diameter would also diminish since m is indeed defined for this ring. Thus, a ring disappears each time d is reduced by $\lambda/2$. At d = 0 the circular fringe pattern disappears. If the reflecting planes of mirrors M_3 and M_4 are not parallel in the sense of Fig. 2, one obtains curved fringes, which change into straight fringes at d = 0.

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Equipment

Position	Material	Item No.	Quantity
1	Optical breadboard, optimized Damping	08751-00	1
2	YAG Laser green, 1 mW, 532 nm for component holder	08767-99	1
3	Surface mirror 30 x 30 mm	08711-01	4
4	Accesory set for optical base plate	08750-50	2
5	Holder for diaphragms and beam splitters beam height 15cm	08719-01	1
6	Beam splitter 1/1, non polarizing	08741-00	1
7	Lens made of glass, biconvex, f = + 20 mm	08059-00	1
8	Component holder	08043-00	2
9	Screen, white, 150x150 mm	09826-00	1
10	Interferometer plate with precision drive	08715-01	1
11	Photoelement	08734-00	1
12	PHYWE Digital multimeter, 600V AC/DC, 10A AC/DC, 20 MΩ, 200 µF, 20 kHz, −20°C…760°C	07122-00	1
13	Measuring tape, l = 2 m	09936-00	1
14	Adjusting support 35 x 35 mm	08711-04	4
15	Ring for component holder	08044-00	2

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

PHYWE

Setup and Procedure

Setup (1/3)

PHYWE

In the following, the pairs of numbers in brackets refer to the co-ordinates on the optical base plate in accordance with Fig. 3. These co-ordinates are only a rough guideline. Perform the experimental set-up according to Fig. 3. The recommended set-up height (beam path height) is 130 mm.

- The lens L [1,7] must not be in position when making the initial adjustments. Before beginning with the adjustment, mount the extra plate P (fine adjustment drive on a plate) on the optical base plate according to Fig.3 ;whereby the co-ordinate Iines should coincide as precisely as possible.

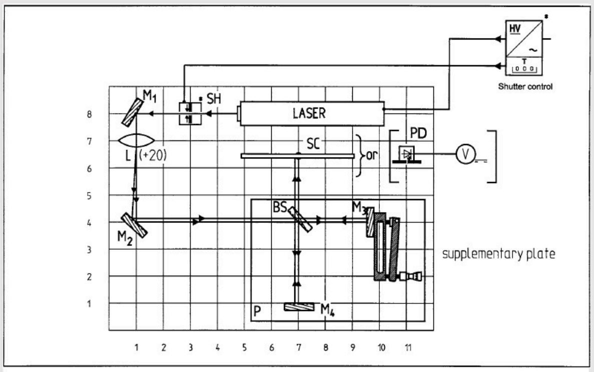


Fig. 3: Experimental setup

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Setup (2/3)

PHYWE

- When adjusting the beam path with the adjustable mirrors M_1 [1,8] and M_2 [1,4], the beam is aligned with the 4th y co-ordinate of the base plate.
- Adjust the mirror M3 [10,4], initially without the beam splitter BS [7,4], such that the reflected beam strikes the same point on mirror M_2 from which it previously originated.
- Now, place the beam splitter BS with its metallized side facing mirror M_2 in the beam path in such a manner that a partial beam strikes mirror M_3 unchanged and the other partial beam strikes mirror M_4 [7,1] perpendicularly along the 7th x co-ordinate of the base plate.

Setup (3/3)

PHYWE

- The beam which is reflected by mirror M_4 must now be adjusted with the adjusting screws such that it strikes the same point on screen SC [7,6.5] as the partial beam that originated at mirror M3 and was subsequently reflected by the beam splitter BS [7,4]. A slight flickering of the luminous points which have been made to coincide indicates nearly exact adjustment.
- By placing the lens L [1,7] in the beam path the luminous points are expanded.
- Now observe the interference patterns on screen SC (stripes, circles).
- By meticulously readjusting the mirrors M_3 and M_4 using the adjusting screws, one obtains concentric cireies.

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Procedure (1/4)

On determining the wavelength of the laser light:

- To perform this measurement, the path distance between the mirror M_3 and the beam splitter BS must be changed. In the process, the position of mirror M_3 is altered using a lever arm (lever transmission ratio approx. 20:1) and a micrometer screw (2 turns correspond to 1 mm), and thus the optical path length of the light beam is also changed.
- On changing the optical path lengths, one sees changes in the centre of the interference rings from maxima to minima and visa versa. Whether the path length increases or decreases becomes apparent in the following: for decreasing path length, the centre represents a source of maxima and minima; or for increasing path lengths it is a sink for the interference maxima and minima.
- According to the theory a change from minimum to minimum occurs when the optical path length A·d is changed by A, Le. in the set-up used the distance between the beam splitter BS and the mirror M_3 changes by A/2.

Procedure (2/4)

- To determine the wavelength of laser light, the changes in the distance between M_3 and BS are measured (by reading the initial and final values on the micrometer screw) and the number of changes from minimum to minimum (or maximum to maximum) are counted.

On recording the contrast function:

- In this case, the screen SC is replaced by a photo cell PD for the determination of the contrast function K. To ensure that the photocell does not measure the intensity across different maxima and minima of the circular interference fringes, reduce the size of the slotted diaphragm with black tape such that only a small aperture of approximately 1 mm^2 remains in the middle.
- For this part of the experiment, make the room as dark as possible to keep the dark current of the photocell as low as possible.

Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Procedure (3/4)

PHYWE

- To determine the contrast function, measure the intensities of minima and maxima at varying the optical separation of the mirrors. Change the separation using only mirror M_4. This mirror is only to be moved along the 7th x co-ordinate.
- Measure the distance between mirrors and beam splitter with a measuring tape. During the repositioning procedure, the mirror must be readjusted at each new position (if necessary, initially without the lens, see above) such that the interference fringes again become visible.
- To measure the intensities of minima and maxima, alter the position of the mirror M_3 slightly using the micrometer screw so that one can see which minimal and maximal voltage values can be measured with the multimeter (measuring range approx. 500 mV).

Procedure (4/4)

PHYWE

- The difference in optical path length between the two mirrors and the beam splitter should be varied between 0 and 10 cm: I.e., when the distance from M_3 to the beam splitter is approximately 13 cm, mirror M_4 should be at position [7,2.25] at its minimum distance of approximately 8 cm from the beam splitter BS and at position [7,1.25] at its maximum separation of circa 13 cm from it.
- In the process, one must take into consideration that the larger the separation differences are, the smaller the radii of the circular interference fringes are. Consequently, at large separation differences the measurement of the maximum and minimum intensities is uncertain and, as a result of the relatively large diaphragm aperture, subject to large errors.

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Evaluation

Evaluation (1/12)

To measure the wave length of the light, count the circular fringe changes while moving the mirror with the micrometer screw (transmission ratio approx. 20:1).

In the process, a shift of the mirror by 43.157 μm is measured and N = 135(1) circular fringe changes are counted.

$$\lambda = \frac{2 \cdot d}{N} = \frac{2 \cdot 43.157}{135} \mu\text{m}$$

From these values, the wavelength of light λ = 639(10) nm is obtained.

Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Evaluation (2/12)

PHYWE

The temporal coherence - the coherence time and length - of a laser can be determined with the aid of a Michelson interferometer.

As a result of the different optical distances which the light traverses in the interferometer, the laser light which has been split into two beams undergoes a temporal retardation τ and is then caused to interfere with itself. The coherence time τ_c is the lag time at which the wave trains are still capable of interference, thus

$\tau < \tau_c$ (5)

The coherence length is therefore:

$l_c = c \cdot \tau_c$ (6)

where c : speed of light.

Evaluation (3/12)

PHYWE

The laser's resonator determines the possible oscillation modes via its resonance condition $L_R = n \cdot \lambda/2 (n = 1, 2, 3...)$. In the process, only those frequencies which Iie within the natural emission spectrum of the amplification medium and above the threshold of resonator losses occur (see Fig.4).

The contrast between the bright and dark circular fringes of the interference patterns is a measure of the interference capability of light. This can be determined by applying the autocorrelation function of light.

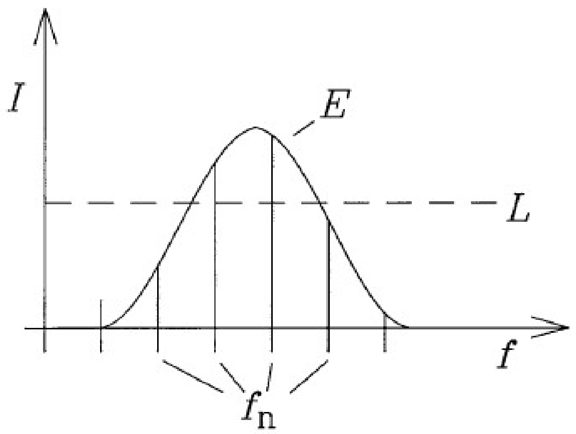


Fig. 4: Frequency spectrum of a laser. With Emission spectrum E, the resonator modes f_n and the laser threshold L

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Evaluation (4/12)

PHYWE

If $E(r,t) = A \cdot e^{i(kr-\omega t+\varphi)}$ is the complex electrical field vector of the light wave at location rand at time t, it follows that the intensity I (except for a constant factor) is:

$I = (E \cdot E^*)$ (7)

where E* is the conjugated complex vector of E and < > is a temporal average.

In our case the following results for the two waves E_1 and E_2 in the Michelson interferometer:

$I_{\text{res}} = ((E_1 + E_2) \cdot (E_1 + E_2)^*) = I_1 + I_2 + 2 \cdot Re(E_1 \cdot E_2^*)$ (8)

In our experiment E_1 and E_2 are identical in the ideal case, except for the temporal shift τ, therefore:

$E_2(t) = E_1(t+\tau)$ (9)

Evaluation (5/12)

PHYWE

$\Gamma(\tau) = (E(t) \cdot E^*(t+\tau))$ (10)

$\Gamma(\tau)$ is the autocorrelation function or also the Self-coherence function of light in this case.

For the resulting intensity, the following is thus obtained:

$I_{\text{res}} = 2I + 2Re(\Gamma(\tau))$ (11)

The contrast in the interference pattern is given by:

$K = \frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}$ (12)

The standardised self-coherence function is the complex degree of self-coherence:

$\gamma(\tau) = \frac{\Gamma(\tau)}{\Gamma(0)}$ (13)

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Evaluation (6/12)

PHYWE

so that with (9) and (10) the following results:

$E_1 = A_1 \cdot e^{i(k_1 x - \omega_1 \tau)};\ E_2 = A_2 \cdot e^{i(k_2 x - \omega_2 t)}$ (14)

and therefore $K = |\gamma(\tau)|$

For an ideal planar monochromatic wave in the x direction, the following contrast function would result:

$K = |\gamma(\tau)| = 1$

for $E = A \cdot e^{i(kx - \omega t)}$ $\text{with} \gamma(\tau) = e^{-i\omega\tau}$. This means that the coherence time and length would be infinitely long in the ideal case for a single frequency. However, in reality, the coherence length is limited by the naturalline width (in gas lasers primarily additionally by Doppler broadening) of the spectral lines.

Evaluation (7/12)

PHYWE

If a laser oscillates in two modes having the frequencies ω_1 and ω_2 with

$I_{\text{max}} = 2 \cdot | + 2| \cdot |\gamma(\tau)|;\ I_{\text{min}} = 2 \cdot | - 2| \cdot |\gamma(\tau)|$

the coherence function is the following:

$\Gamma(\tau) = ((E_1(t) + E_2(t)) \cdot (E_1(t+\tau) + E_2(t+\tau))*)$

$= \Gamma_1(\tau) + \Gamma_2(\tau)$

$= |A_1|^2 e^{-i\omega_1 \tau} + |A_2|^{2e^{-i}\omega_2 \tau}$ (15)

and the degree of self-coherence is given by:

$\gamma(\tau) = \frac{|A_1|^2}{|A_1|^2 + |A_2|^2} e^{-i\omega_1 \tau} + \frac{|A_2|^2}{|A_1|^2 + |A_2|^2} e^{i\omega_2 \tau}$ (16)

PHYWE

Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Evaluation (8/12)

For simplification's sake A_1 and A_2 are equal. With $\omega_2 = \omega_1 + \Delta\omega$ we obtain the following results for the contrast of the interference grating:

$K = |\gamma(\tau)|$

$= \sqrt{\frac{1}{2}(1+\cos(\Delta\omega\tau))}$ (17)

$= |\cos(\frac{\Delta\omega\cdot\tau}{2})|$

If the 5-mW laser is used, the following frequency separation of the axial modes results for a resonator length L of approximately 30 cm:

$\Delta\omega = 2\pi\cdot\Delta f = 2\frac{\pi\cdot c}{2L}(= 3.1\ \mathrm{GHz})$ (18)

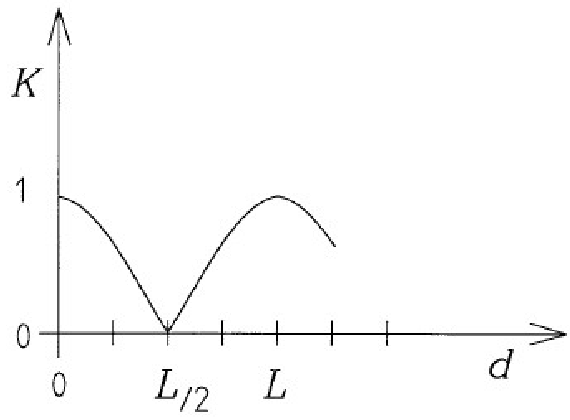


Fig. 5: Theoretical contrast function K of a 2-mode laser.

Evaluation (9/12)

The propagation delay time T results from the mirror shift d:

$\tau = \frac{2d}{c}$ (19)

With (18) and (19) the following results for the contrast function K:

$K = |\cos(\frac{2\cdot\pi\cdot d}{2\cdot L})| = |\cos(\frac{\pi\cdot d}{L})|$ (20)

For the 5-mW laser the following is valid:

- according to (18), the mode separation Δf is obtained.
- with $K = \frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}} = |\cos(\frac{\Delta\omega\cdot\tau}{2})|$

Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Evaluation (10/12)

one obtains for the critical delay time τ_c:

$K = 0 = |\cos(\frac{\Delta\omega\cdot\tau_c}{2})|$ or $\frac{\pi}{2} = \frac{2\pi\cdot\Delta f\cdot\tau_c}{2} \rightarrow \tau_c \approx 1\ \mathrm{ns}.$

- This results in a minimum of the contrast function (20) at:

$o = |\cos(\frac{\pi\cdot d}{L})|$ or $\frac{\pi}{2} = \frac{\pi\cdot d}{L} \rightarrow d = \frac{L}{2} \approx 15.0\ \mathrm{cm}.$

The experimental data for the contrast function K as a function of the mirror shift d are given in Fig. 6.

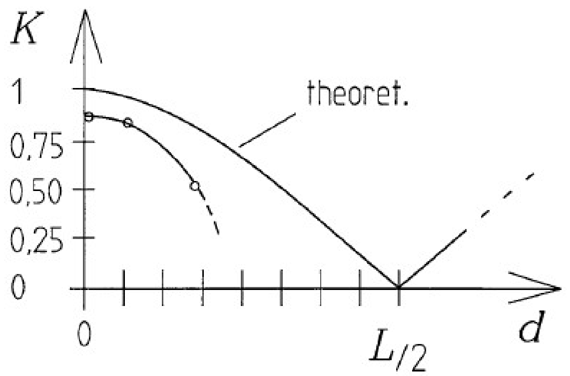


Fig. 6: Experimentally determined contrast function in comparison to the theoretical contrast function K of a 2-mode laser.

Evaluation (11/12)

It becomes apparent that the theoretical maximum is not reached; this can be due to several factors:

1. If the mirrors are inadequately parallel, the contrast function only reaches a smaller maximum value (thus, does not reach 1) and a larger minimum value (hence, does not drop to zero), see Fig. 7.
2. The division of the beam splitter is not ideal, Le. 50:50. Therefore, the derivation of the contrast function would have had to have been modified.
3. The 5-mW laser does not only oscillate in 2 modes, but rather the amplification is sufficient to allow the laser to oscillate in three axial modes: this shortens the coherence time. (A spectral analysis has established that a third mode is only possible to a considerably lesser degree than the two primary axial modes!)
4. The aperture in front of the photodiode was not made small enough. Consequently, it covers and averages different intensity regions.

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Evaluation (12/12)

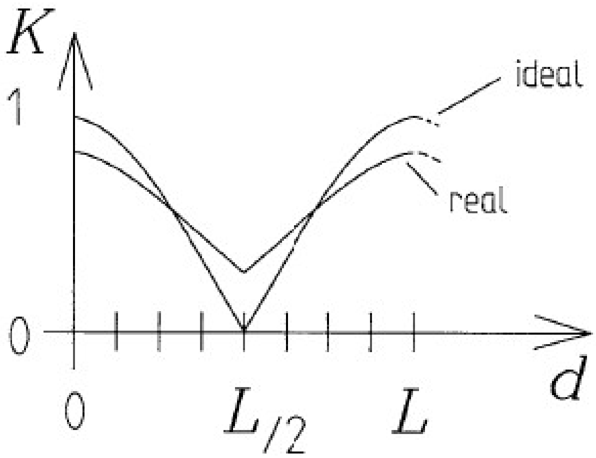


Fig. 7: Theoretical contrast function K of 2-mode laser under ideal and real conditions.

No.	I_{min} [mV]	I_{max} [mV]	d [cm]	$K_{acc.}$
1	20	220	0.2	0.835
2	23	214.7	2.0	0.8065
3	67	200.5	5.3	0.4990

Table 1: Experimental data

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de