Diffraction intensity of multiple slits and grids

(Item No.: P2230401)

Curricular Relevance

Area of Expertise: Physics
Education Level: University
Topic: Light and Optics
Subtopic: Diffraction and Interference
Experiment: Diffraction intensity of multiple slits and grids

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	10 Minutes	20 Minutes	2 Students

Additional Requirements:

- PC/Laptop with at least Windows XP

Experiment Variations:

Keywords:

Huygens principle, interference, Fraunhofer- and Fresnel diffraction, coherence, laser

Introduction

Overview

Principle

Multiple slits which all have the same width and the same distance among each other, as well as transmission grids with different grid constants, are submitted to laser light. The corresponding diffraction patterns are measured according to their position and intensity.

Fig. 1: Experimental set-up to investigate the diffraction intensity of multiple slits and grids.

Equipment

Position No.	Material	Order No.	Quantity
1	Diodelaser, green, 1 mW, 532 nm	08764-99	1
2	Digital array camera	35612-99	1
3	Optical bench expert, l = 1500 mm	08281-00	1
4	Base for optical bench expert, adjustable	08284-00	2
5	Slide mount for optical bench expert, h = 30 mm	08286-01	5
6	Lens holder, beam height 120 mm	08012-01	2
7	Object holder, 5x5 cm	08041-01	1
8	Lens, mounted, f +20 mm	08018-01	1
9	Lens, mounted, f +100 mm	08021-01	1
10	Diaphragm, 3 single slits	08522-00	1
11	Diaphragm, 4 multiple slits	08526-00	1
12	Diffraction grating, 4 lines/mm	08532-00	1
13	Diffraction grating, 8 lines/mm	08534-00	1
14	Diffraction grating,10 lines/mm	08540-00	1
15	Diffraction grating,50 lines/mm	08543-00	1
16	Diaphragm, 4 double slits	08523-00	1
17	Screen, white, 150x150 mm	09826-00	1

Tasks

1. The position of the first intensity minimum due to a single slit is determined, and the value is used to calculate the width of the slit.
2. The intensity distribution of the diffraction patterns of a threefold, fourfold and fivefold slit, where the slits all have the same widths and the same distance among each other, is to be determined. The intensity relations of the central peaks are to be assessed.
3. For transmission grids with different lattice constants s, the position of the peaks of several orders of diffraction k is to be determined. This value is used to calculate the wavelength λ of the laser.

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Set-up and procedure

The experimental set-up is shown in Fig. 1, and suggested positions on the optical bench are listed in Table 1. With the assistance of the $f = 20\,\mathrm{mm}$ and $f = 100\,\mathrm{mm}$ lenses, a widened and parallel laser beam is generated, which must impinge centrally on the digital array camera set as far behind the slit as possible. The diffracting objects are set in the object holder, and the diffracting object under investigation must be set vertically in the object holder and uniformly illuminated. Darken the room when performing experiments.

Table 1: Positions on the optical bench.

Material	**Position** (cm)
Laser	2
Lens, f +20 mm	14
Lens, f +100 mm	27
Diffracting objects	33

Caution: Never look directly into a non attenuated laser beam

The diffraction intensity value ranges are determined for the multiple slits using the digital array camera. Connect the camera to the USB port of the PC, and the software should install automatically. For further information click the "Help" button in the bottom right. For the transmission grids, the positions of diffraction peaks must be determined so as to be able to calculate the wavelength of the laser. Results will appear noisier than those shown in Figs. 2 and 3.

Theory and evaluation

An optical lattice is a periodic structure consisting of N parallel single slits for the diffraction of light. s is the lattice constant (the space between slits measured from center to center) and b is the width of a single slit.

Incident light is diffracted by the structures which are in the order of the wavelength λ of the radiation, so that spherical waves are arising.

If a slit width b considered against the wavelength is small, only one elementary wave is transmitted per slit. As the slit width of the grids during this experiment are large compared to the wavelength λ, this approximation can not be used.

The interference pattern of monochromatic light of wavelength λ behind a lattice can be described as a superposition of the Huygens spherical waves of each slit.

The path difference d_1 of the marginal rays of a slit of width b is

$$d_1 = b \cdot \sin(\varphi).$$

This results in a phase difference of

$$\delta_1 = \frac{2\pi \cdot d_1}{\lambda} = \frac{2\pi \cdot b \cdot \sin(\varphi)}{\lambda}. \qquad (1)$$

The beams of two slits have a path difference of

$$d_2 = s \cdot \sin(\varphi).$$

This results in a phase difference of

$$\delta_2 = \frac{2\pi \cdot d_2}{\lambda} = \frac{2\pi \cdot s \cdot \sin(\varphi)}{\lambda}. \qquad (2)$$

For N beams to be deflected to an observation point at the angle of diffraction φ, is obtained with the amplitude E_φ of the diffracted beam and geometric considerations the following dependence of the intensity (the intensity is proportional to the square of the field strength):

$$I_\varphi \propto \frac{\overline{E}_\varphi^2 \cdot \sin^2(N \cdot \delta_2/2)}{\sin^2(\delta - 2/2)}. \qquad (3)$$

In this case, however, E_φ^2 is the intensity of the beam that is diffracted in front of a single slit in the φ direction. Calculations for a single slit follows:

$$\overline{E}_\varphi^2 \propto \frac{\sin^2(\delta - 1/2)}{(\delta_1/2)^2}. \qquad (4)$$

The diffraction intensity of the entire grid is obtained by plugging in equation (4) in eq. (3):

$$I_\varphi \propto \frac{\sin^2\left(\frac{\pi \cdot b}{\lambda} \cdot \sin\varphi\right)}{\left(\frac{\pi \cdot b}{\lambda} \cdot \sin\varphi\right)^2} \cdot \frac{\sin^2\left(\frac{N \cdot \pi}{\lambda} \cdot s \cdot \sin\varphi\right)}{\sin^2\left(\frac{\pi}{\lambda} \cdot s \cdot \sin\varphi\right)} = \frac{\sin^2\left(\frac{\delta_1}{2}\right)}{\left(\frac{\delta_1}{2}\right)} \cdot \frac{\sin^2\left(\frac{N\delta_2}{2}\right)}{\sin^2\left(\frac{\delta_2}{2}\right)}. \qquad (5)$$

The first part of the product from (5) is thus the intensity distribution of the single slit, and the second part is the result of the interaction of the N slits. It becomes obvious that the minima of the single slits is also retained in the grid, because the first factor becomes zero, so the product is also zero.

According to Fraunhofer the minima and maxima of a single slit are referred to as first class interference, while the interactions of several slits lead to second class interference.

The observation of a single slit (first factor) results in an intensity minimum when the numerator from (5) is zero. In this case:

$$\sin\varphi_h = \frac{h \cdot \lambda}{b}; \quad (h = 1, 2, 3, \ldots). \qquad (6)$$

The angular position of the 1st class peaks is given approximately through:

$$\sin\varphi_h = \frac{2h+1}{2} \cdot \frac{\lambda}{b}; \quad (h = 1, 2, 3, \ldots). \qquad (7)$$

If several slits act together, the minima of the single slits always remain. Supplementary 2nd class minima appear when the 2nd factor also becomes zero.

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For a double slit ($N=2$), the zero points can be easily calculated after application of an addition theorem to the second factor of equation 5 ($N=2$) from the following conditional equation:

$$4\cos^2(\frac{\pi}{\lambda}\cdot\sin\varphi)=0. \quad (8)$$

This term is zero for

$$\sin\varphi_k=\frac{2k+1}{2}\cdot\frac{\lambda}{s}; \quad (k=1,2,3,\ldots). \quad (9)$$

It is easily seen to equation (5), the second term oscillates faster since $s>b$ and thus $\delta_2>\delta_1$ and $N>1$ (see Fig. 2). For the intensity I of the second-order principal maxima applies in addition to the grid, as the second factor of (5) is apparent:

$$I\propto N^2. \quad (10)$$

The main 2nd class peaks, therefore, become more prominent as the number of slits increases. There still are ($N-2$) secondary 2nd class peaks between the main peaks.

When light is diffracted through transmission grids with lattice constant s, the diffraction angle φ of the main peaks fulfills the following relation:

$$\sin\varphi_k=\frac{k\&cdot\lambda}{s}; \quad (k=1,2,3,\ldots). \quad (11)$$

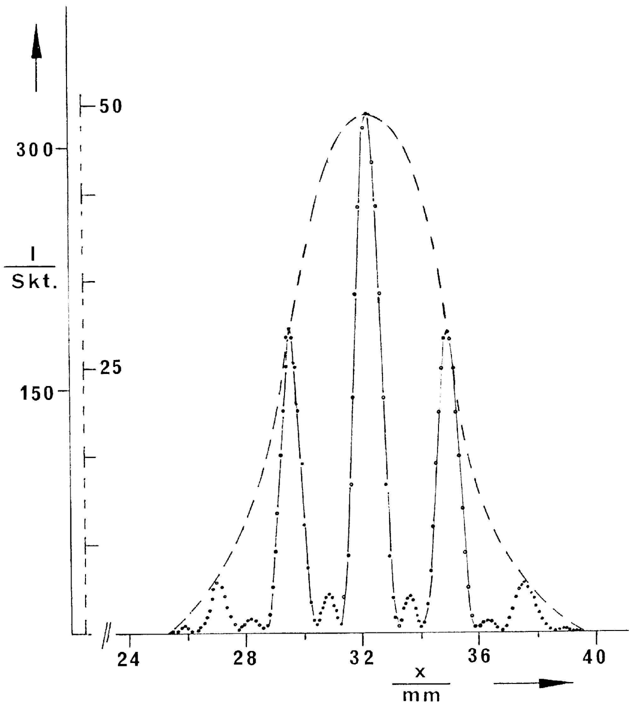

Fig. 2: Diffraction intensity I as a function of the position x for a threefold slit, b_1 = 0.1 mm and s = 0.25 mm. Distance between threefold slit and detector: L = 107 cm. For comparison, the intensity distribution of a single slit b = 0.1 mm, is plotted as a dotted line.

In Fig. 3 the diffraction intensity I for a threefold slit is plotted against the position x of the detector..

On display are five main peaks (0., ± 1. and ± 2. order) second class with the $N-2=1$ intermediate secondary maxima.

The slit width b is expressed in the envelope which is adapted in the figure on the ordinate: This represents the maximum 0th order of the first class.

According to (5), it can be seen that the envelope (i.e. the interference 1st class) the interference pattern more strongly attenuates by the lattice periodicity to increasing orders, the wider the single slit b. One obtains the width of the slits $b_1=0.097\,\mathrm{mm}$ from (6), with the distance between the two 1st class minima, so the envelope (\\(\sin\varphi \approx \tan\varphi, \, L = \, 107 \, \mathrm{cm}, \\)).

Fig. 3 shows the diffraction figure of a fourfold slit. In this case, the number of 2nd class peaks is $(N-2)=2$. In the same way, diffraction through a fivefold slit (no figure) yields $(N-2)=3$ 2nd class secondary peaks.

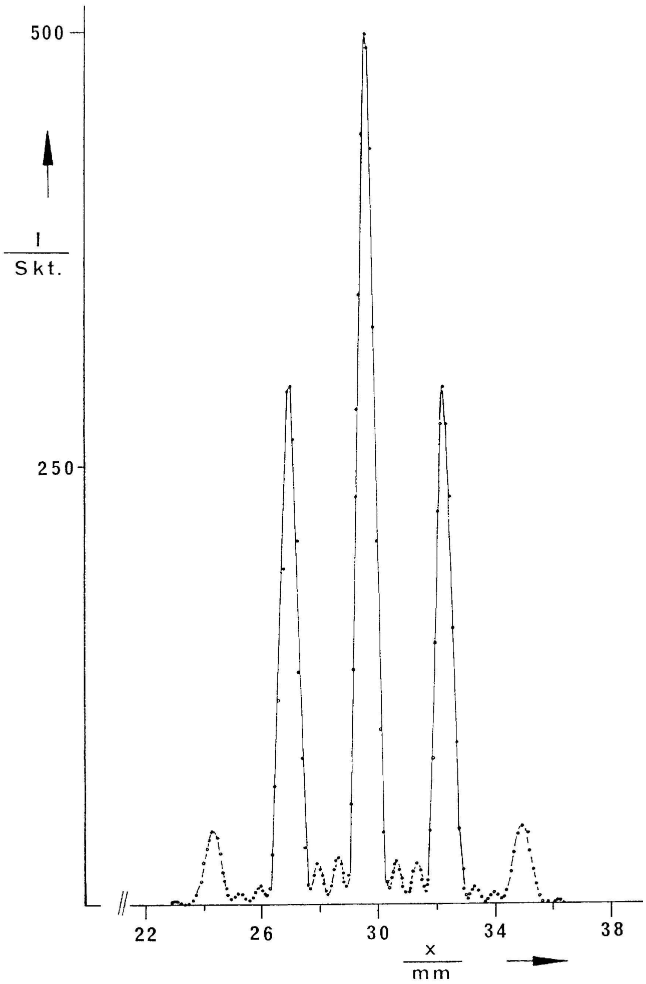

Fig. 3: Diffraction intensity I as a function of the position x for a fourfold slit with b_1 = 0.1 mm and s = 0.25 mm

Table 2 gives the intensity values of the central peaks of the diffracting objects with $N=3$ to $N=5$, as well as the relative values determined empirically and according to equation (10).

Table 1: Intensity values of central peaks of diffracting objects with $N=3$ to $N=5$.

	exp.	**theor.**
$I_{05}\ (N=5)=720\,Skt.$		
$I_{04}\ (N=4)=500\,Skt.$	$I_{05}/I_{04}=1.44$	$(5/4)^2=1.56$
$I_{03}\ (N=3)=300\,Skt.$	$I_{05}/I_{03}=2.40$	$(5/3)^2=2.78$

Fig. 4 shows the distances Δx between the transmission maximum ($k=0$) and the diffraction peaks measured for 4 different transmitting grids up to the 3rd order ($k=3$) as a function of the lattice constant s. With (11), Fig. 4 yields $\lambda=532\,\mathrm{nm}$ as an average value for the wavelength of the used laser light.

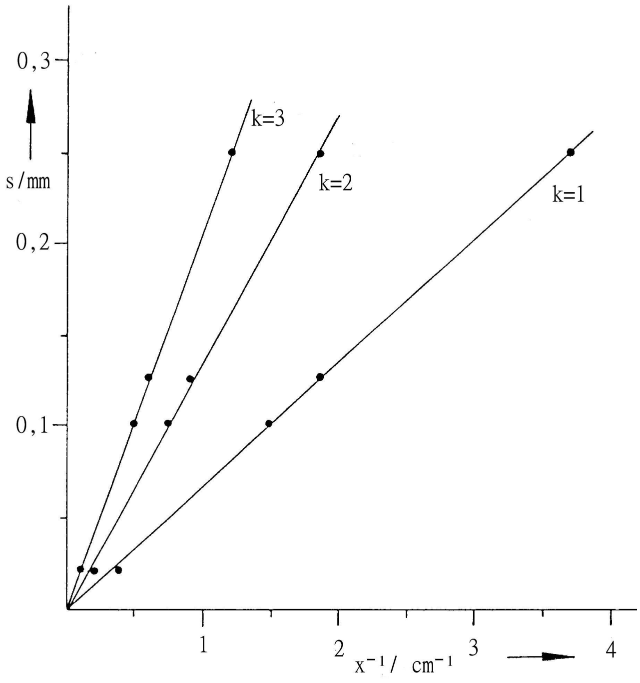

Fig. 4: Reciprocal distance of the diffraction peaks up to the 3rd order of diffraction (k = 3) as a function of the lattice constant.

Plotting the lattice constants of the reciprocal spacing of the diffraction maxima results in straight lines.