

# Capacitance of metal spheres and of a spherical capacitor



Physics

Electricity &amp; Magnetism

Simple circuits, resistors &amp; capacitors



Difficulty level

hard



Group size

2



Preparation time

10 minutes



Execution time

20 minutes

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<http://localhost:1337/c/60088197e5fe6b0003958e25>

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# General information



## Application

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Fig. 1 : Experimental set-up to determine the capacitance of conducting spheres.

Capacitors are a widely used component of electronics. As such it is very important to understand the specific capacities of differing shapes.

## Other information (1/2)

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**Prior****knowledge****Main****principle**

The prior knowledge for this experiment is found in the Theory section.

Metal spheres with different radii and a spherical capacitor are charged by means of a variable voltage. The induced charges are determined with a measuring amplifier. The corresponding capacitances are deduced from voltage and charge values.

## Other information (2/2)

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**Learning  
objective****Tasks**

The goal of this experiment is to investigate the capacitance of spherical capacitors.

1. Determination of the capacitance of three metal spheres with different diameters.
2. Determination of the capacitance of a spherical capacitor.
3. Determination of the diameters of each test body and calculation of their capacitance values.

## Theory (1/2)

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The capacitance  $C$  of a sphere with radius  $R$  is given by:

$$C = 4\pi\epsilon_0 R \quad (1)$$

Using (2), the voltage values  $U_1$ , which were determined by means of the measuring amplifier, allow to determine the corresponding charge value  $Q$ :

$$Q = (C_{co} + C_{ca})U_1 = (C_{co} + C_{ca})\frac{U}{V}$$

$$\text{with } C_{ca}U_1 = \frac{U}{V} \quad (2)$$

$C_{co}$  = capacitance of the conductors;  $C_{ca}$  = capacitance of the parallel capacitor,  $U$  = displayed voltage,  $V$  = amplification factor,  $U_1$  = measured voltage

## Theory (2/2)

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On the other hand, the charge  $Q$  of the conductor is:

$$Q = C_{co}U_2 \quad (3)$$

Finally, charges may be calculated using (2) and (3).

$$U_1/U_2 = C_{co}/C_{ca}$$

The capacitance  $C$  of a spherical capacitor is given by

$$C = 4\pi\epsilon_0 \frac{r_1 r_2}{r_2 - r_1} \quad (4)$$

( $r_1$  = Radius of the interior sphere;  $r_2$  = Radius of the exterior sphere)

## Equipment

Position	Material	Item No.	Quantity
1	Conductor ball, d 20mm	06236-00	2
2	Conductor ball, d 40mm	06237-00	1
3	Conductor ball, d 120mm	06238-00	1
4	Hemispheres, Cavendish type	06273-00	1
5	Hollow plastics ball, w. eyelet	06245-00	1
6	Capillary tube, straight, l 250mm	36709-00	1
7	Copper wire, d = 0.5 mm, l = 50 m	06106-03	1
8	Insulating stem	06021-00	2
9	High-value resistor, 10 MOhm	07160-00	1
10	PHYWE High voltage power supply with digital display, 10 kV DC: 0... ± 10 kV, 2 mA	13673-93	1
11	Capacitor 10nF/ 250V, G1	39105-14	1
12	PHYWE Universal measuring amplifier	13626-93	1
13	Analog multimeter, 600V AC/DC, 10A AC/DC, 2 MΩ, overload protection	07021-11	1
14	Digital multimeter, 600V AC/DC, 10A AC/DC, 20 MΩ, 200 μF, 20 kHz, -20°C... 760°C	07122-00	1
15	Connecting cord, 30 kV, 1000 mm	07367-00	1
16	Screened cable, BNC, l = 750 mm	07542-11	1
17	Adaptor, BNC socket/4 mm plug	07542-20	1
18	Connector, T type, BNC	07542-21	1
19	Adapter, BNC male/4 mm female pair	07542-26	1
20	Vernier calliper, plastic	03011-00	1
21	Barrel base expert	02004-00	2
22	Support base DEMO	02007-55	1
23	Right angle clamp expert	02054-00	4
24	Support rod, stainless steel, 750 mm	02033-00	1
25	Support rod, stainless steel, 500 mm	02032-00	1
26	Universal clamp with joint	37716-00	1
27	Alligator clip, insul., strong, 10 pcs	29426-03	1
28	Connecting cord, 100 mm, green-yellow	07359-15	1
29	Connecting cord, 32 A, 750 mm, green-yellow	07362-15	2
30	Connecting cord, 32 A, 500 mm, blue	07361-04	2
31	Connecting cord, 32 A, 500 mm, red	07361-01	2

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# Setup and Procedure

## Setup (1/3)

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The experimental set-up to determine the capacitance of spherical conductors is shown in Fig. 1. Fig. 2 only shows the part of the experimental set-up which must be modified in order to determine the capacitance of a spherical capacitor. The spherical conductor ( $d = 2 \text{ cm}$ ) held on a barrel base and insulated against the latter is connected by means of the high voltage cord over the  $10 \text{ M}\Omega$ - protective resistor to the positive pole of the  $10 \text{ kV}$  output of the high voltage power supply. The negative pole is earthed. This sphere is briefly brought into contact with the test spheres to charge it. High voltage always must be reset to zero after charging. After every measurement, the charging voltage is increased by  $1 \text{ kV}$ . Before being charged anew, the test spheres must be discharged through contact with the free earth connecting cable. The charges of the test spheres are determined with a measuring amplifier. The high-resistance input of the electrometer is used for this. An auxiliary  $10 \text{ nF}$  capacitor is connected in parallel to the BNC test cord fitted with the adapter required to take over the charge. The capacitances of the spherical conductors are determined from the voltage and charge values; this is done using the average calculated over a number of charge measurement values.

**Never apply high voltage to the amplifier input.**

## Setup (2/3)

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To determine the capacitance of a spherical capacitor, the experimental set-up is altered as shown in Fig. 2. The Cavendish hemispheres are put together so as to form a complete sphere with a small circular orifice at the top. The plastic sphere with conducting surface is suspended from a copper wire in the centre of the sphere. The copper wire is lead through a glass capillary tube which is wrapped in earthed aluminium foil to neutralise stray capacitances (Fig. 3). The aluminium foil may not touch the hemispheres. The interior sphere must be connected to the central socket of the high voltage power supply. This is done by means of a crocodile clip over the high voltage cord, before which a 10 M $\Omega$  protective resistor is connected.

The lower socket is earthed again. Voltage is increased in steps of 100 V and may not increase above 1000 V for the safety of the digital multimeter. The corresponding mean values of the charges are read determined for the hemispheres similarly as described in Part. After every measurement, the hemispheres must be discharged with the free earthing cord. Whilst doing this, it must be assured that no high voltage is induced.

## Setup (3/3)

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Fig. 2 : Part of the experimental set-up used to determine the capacitance of a spherical capacitor.

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# Evaluation

## Task 1 (1/2)

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Using (1), the capacitance of the conducting spheres may be calculated:

Sphere ( $R = 0.121 \text{ m}$ ):  $C = 6.7 \cdot 10^{-12} \text{ As/V} = 6.70 \text{ pF}$

Sphere ( $R = 0.041 \text{ m}$ ):  $C = 2.28 \cdot 10^{-12} \text{ As/V} = 2.28 \text{ pF}$

Sphere ( $R = 0.021 \text{ m}$ ):  $C = 1.22 \cdot 10^{-12} \text{ As/V} = 1.22 \text{ pF}$

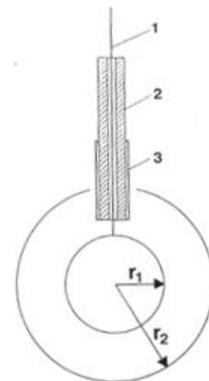


Fig. 3: Sketch showing set-up and the attachment of conducting spheres. (1= Copper wire; 2 = capillary tube; 3 = aluminium foil)

## Task 1 (2/2)

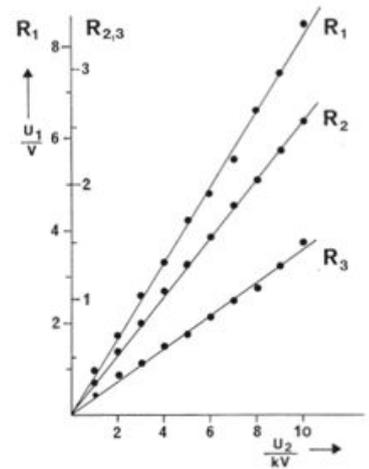
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Fig. 4 shows charging voltage values  $U_2$  as a function of measurement voltage  $U_1$  for the different conducting spheres. With the slope of the lines and knowing  $C_{ca}$ , the capacitances of the conducting spheres are obtained:

$$C(R_1) = 7.55 \text{ pF}; C(R_2) = 72.33 \text{ pF}; C(R_3) = 1.32 \text{ pF};$$

(With the assistance of a measuring bridge, the capacitance  $C_{ca}$  of the parallel capacitor was determined separately to be  $C_{ca} = 9.1 \text{ nF}$ . If exact capacitance determination is impossible, the nominal capacitance value of the capacitor must be used for calculation; this may, however, be affected by design tolerance deviations of  $\pm 10\%$ ).

Fig. 4:  $U_1$  as a function of  $U_2$  measured on conducting spheres with



## Task 2

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With  $r_1 = 0.019 \text{ m}$  and  $r_2 = 0.062 \text{ m}$  for the spherical capacitors, capacitance calculation yields  $C = 3.0 \text{ pF}$ . Fig. 5 once more represents measurement value pairs  $U_1$  and  $r_1$ . According to the assessment procedure described above and using the slope of the graph line, the empirically determined capacitance of the spherical capacitor is found to be  $C = 3.5 \text{ pF}$ . Capacitance values determined experimentally are always higher than the calculated values. This discrepancy is due to unavoidable dispersive capacitances.

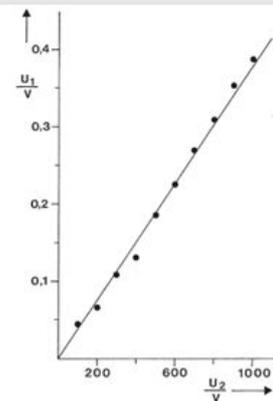


Fig. 5:  $U_1$  as a function of  $U_2$  measured on a spherical capacitor.