RLC circuit

Physics	Electricity & Magnetism	Electronics

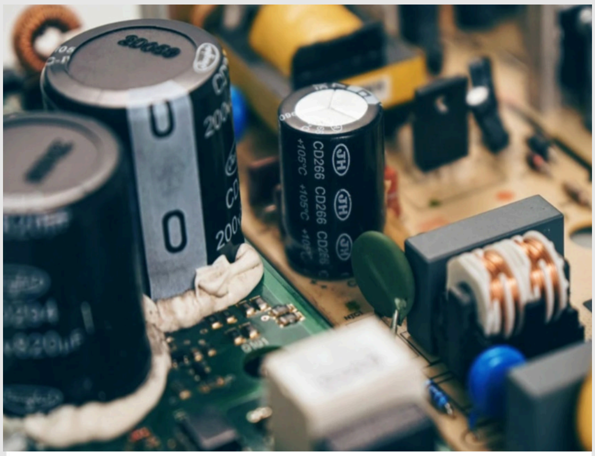
Difficulty level	Group size	Preparation time	Execution time
hard	-	20 minutes	20 minutes

This content can also be found online at:

http://localhost:1337/c/64afda86a2e6800002740834

General information

Application



capacitators, coils and resistors on a board

An electrical oscillating circuit, also called resonant circuit, is a resonant electrical circuit consisting of a coil (inductance L) and a capacitor (capacitance C), which can perform electric current oscillations. A RLC-circuit is an circiut that also contains a damping resistor (resistance R). In this LC oscillating circuit, energy is periodically exchanged between the magnetic field of the coil and the electric field of the capacitor, resulting in alternating high current or high voltage. The resonance frequency is dependend on the inductance and the capacitance included in the circuit. RLC-circuits are often used as frequency filters or resonators in electronic devices, e.g. in radio transmitters and receivers in order to resonate on a special frequency.

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Other information (1/2)

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Prior knowledge

Basic knowledge of physical quantities such as current, voltage and resistance must be available. Furthermore basic knowledge about the process of electromagnetical induction as well as the charging and discharging behaviour of a capacitor should be known. Ideally, previous experience with RC and RL alternating current circuits is already available.

Scientific principle

Combined coils and capacitors in an AC circuit lead to an oscillating current behaviour, which shows a resonance behaviour with a resonance frequency f_0, that depends on the inductance L and the capacitance C in the circuit:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

In an RLC-circuit the resistor induces current damping.

Other information (2/2)

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Learning objective

After the successful completion of this experiment you will be able to theoretically describe the phenomenon of combined inductances and capacitances with respect to alternating currents. You will also be able to experimentally determine the resonant frequencies in dependence of the connected impedances and capacitances.

Tasks

- Measure the voltage drop U over the LC-component and the current I through the circuit and determine the resonance frequency for both combinations of coil and capacitor.
- Determine the impedance Z of the LC-component for both circuits.
- Determine the bandwidth B and Q-factor for both circuits.

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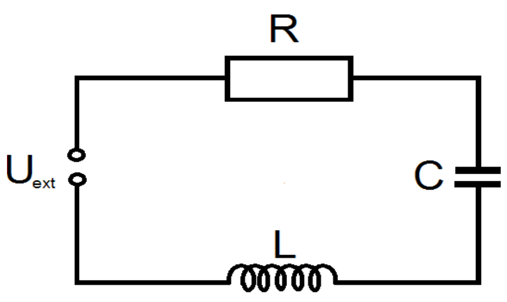
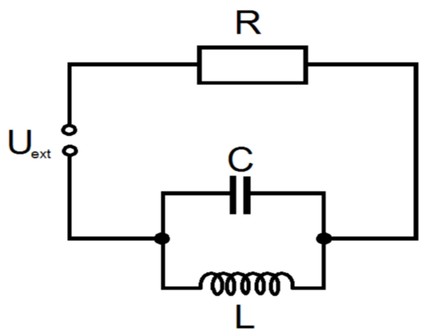
Safety instructions

The general instructions for safe experimentation in science lessons apply to this experiment.

Theory (1/8)

An RLC-circuit consists of a resistance (R), an inductance (L) and a capacitance (C) sometimes it is also refered to as LC-circuit, because the resistor is only used to simulate the loss-resistance of a real circuit. The oscillating frequency is only dependend on the inductance and the capacitance. Generally one differs between two kinds of RLC-circuits, the series- (left) and the parallel-tuned (right) circuit:





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Theory (2/8) - Series circuit

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When a fully charged capacitor is discharged through an inductance coil, the discharge current induces a magnetic field in the coil, which reaches its maximum, when the capacitor is completely discharged. Then, due to the decreasing current, the change in the magnetic field induces a voltage which according to Lenz's law charges the capacitor with reversed polarity. Again the current decreases to zero when the capacitor is completely charged. At this point, the procedure starts again, but with opposite direction of the current.

In absence of any resistance, this charging and discharging processes would oscillate forever – but because of ohmic resistances, which every real circuit posesses, the oscillation is damped and accordingly the amplitudes of current and voltage decrease over time. According to Kirchoff's voltage law (mesh rule) the total voltage in one loop must add to zero or be equal to an external potential:

$$U_L + U_C + U_R = U_{ext}$$

Theory (3/8) - Series circuit

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Differentiating this equation with certain substitutions of the constants one obtains

$$\ddot{I} + 2\delta\dot{I} + \omega_0^2 I = \frac{\omega}{L} U_0 \cdot e^{i(\omega t + \pi/2)}$$

The real part of the solution for the current results in:

$$I = I_0 \cdot cos(\omega t - \varphi) \quad \text{with} \quad I_0 = U_0 / \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \;,\quad \tan(\varphi) = -\frac{1}{R}\left(\omega L - \frac{1}{\omega C}\right)$$

with phase displacement φ. Thereby the resonance point is found at $\omega^2 = \omega_0^2 = (LC)^{-1}$. The impedance is defined by $Z = \frac{U_{eff}}{I_{eff}}$, which leads for the LC-component of the series-tuned circuit to

$$Z_S = \left|\omega L - \frac{1}{\omega C}\right|$$

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Theory (4/8) - Parallel circuit

In the case of the parallel-tuned RLC-circuit, we apply Kirchhoff's first law (junction rule):

$$I_R + I_L + I_C = 0$$

Derivation with respect to time and using the know identities for the voltages of capacitor and coil leads to

$$\ddot{U} + \tfrac{1}{RC}\dot{U} + \tfrac{1}{LC}U = 0$$

Implementing the approach $U(t) = U_0 \cdot e^{i\omega t}$ and after discarding the imaginary part one directly obtains the resonance frequency $\omega_0 = 1/\sqrt{LC}$. To determine the impedance for the parallel tuned LC circuit, one simply uses Kirchhoff's first law with $I_R = I(t)$ and $I(t) = U(t)/Z$ to obtain

$$\frac{U(t)}{Z_P} = \frac{U(t)}{X_L} + \frac{U(t)}{X_C} \quad \Rightarrow \quad \frac{1}{Z_P} = \left|\frac{1}{i\omega L} + i\omega C\right|$$

Theory (5/8) - Parallel circuit

Kirchhoff's law for the complete circuit (regarding the LC-component as one element) leads to

$$U_{ext} = U_R + U_C \quad \Rightarrow \quad U_0 \cdot e^{i\omega t} = RI + Z_{LC}I$$

$(Z_{LC} = Z_P)$. Therefore the solution for the current is, after neglecting the imaginary part,

$$I(t) = I_0 \cos(\omega t + \varphi) \quad \text{with} \quad I_0 = \frac{U_0}{\sqrt{R^2 + \left(\frac{\omega L}{1-\omega/\omega_0}\right)^2}}$$

The phase displacement is given by

$$\tan(\varphi) = \frac{1}{R \cdot (\frac{1}{\omega L} - \omega C)}$$

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Theory (6/8)

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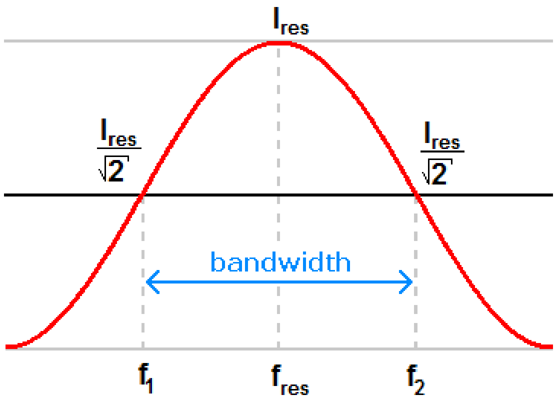

Definition of bandwidth

Comparing the calculations from above, the results are the following. Both circuits (series- and parallel-tuned) have the same resonance frequency

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$$

In the series-tuned case, the impedance tends to zero when the frequency is approaching the resonance frequency, which can be seen in the increase of current. In the parallel-tuned case, the impedance of the LC-component increases while approaching the resonance frequency, which can be seen in the decrease of current.

Theory (7/8)

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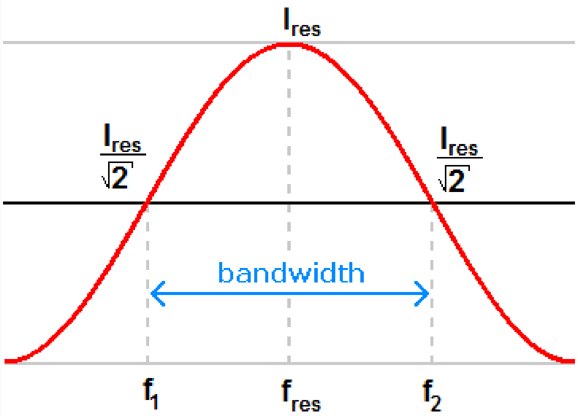

Definition of bandwidth

Another important physical quantity, which describes the behaviour of a resonating system is the bandwith B and the quality factor Q. The bandwidth of a resonance curve is simply defined as the distance between the two points where the maximum current amplitude $A_{max} = A_{res}$ at the resonance drops to a value $\frac{A_{res}}{\sqrt{2}}$, so

$$B = f_2 - f_1$$

The quality factor is thus given by

$$Q = \frac{f_{res}}{B}$$

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Theory (8/8)

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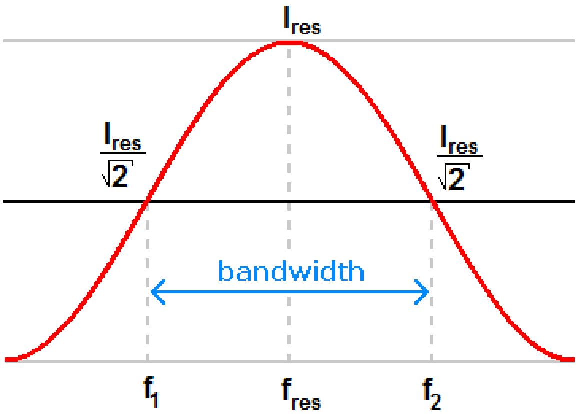

Definition of bandwidth

In the series-tuned circuit, the quality factor is therefore

$$Q_S = \frac{1}{R} \cdot \sqrt{\frac{C}{L}}$$

which can be derived from the equations above (usuall $B = 2\delta$, where δ is the damping. One can see, that the resistor is responsible for the shape of the resonance curve, too. In the parallel-tuned circuit, the quality factor, expressed through the parameters of the electrical components, is given by

$$Q_P = R \cdot \sqrt{\frac{C}{L}}$$

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Equipment

Position	Material	Item No.	Quantity
1	PHYWE Digital Function Generator, USB	13654-99	1
2	Coil, 900 turns	06512-01	1
3	Capacitor 100 nF/250V, G1	39105-18	1
4	Capacitor 470nF/250V, G1	39105-20	1
5	Resistor 47 Ohm, 1W, G1	39104-62	1
6	Resistor 100 Ohm, 1W, G1	39104-63	1
7	Resistor 470 Ohm, 1W, G1	39104-15	1
8	Digitalmultimeter 88C+, 750V AC/DC, 20A AC/DC, 2000MΩ, 200µF, 10 MHz, 20H, -20...1000°C	07029-12	1
9	Connection box	06000-00	1
10	Connecting cord, 32 A, 500 mm, black	07361-05	4
11	Connecting cord, 32 A, 250 mm, black	07360-05	2
12	brigde plug	06027-07	1

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Setup and procedure

Setup (1/3)

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Set up

- Build the electrical circuit according to the shown figures.

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Setup (2/3)

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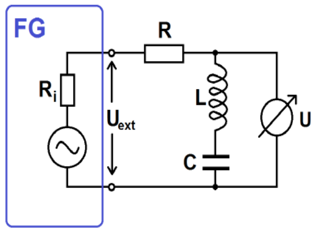
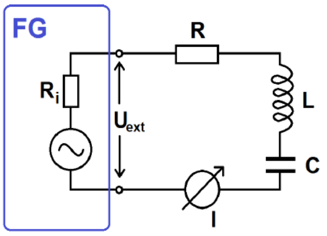
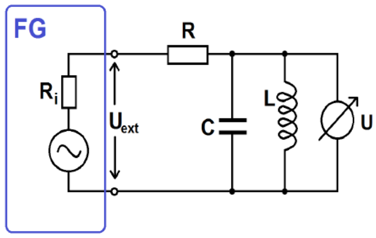
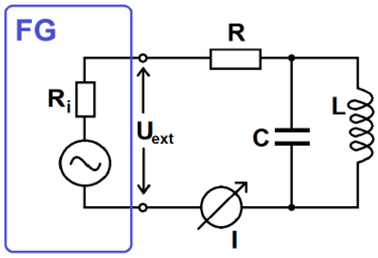




- The experimental set-up for measuring the voltage and current in the series-tuned circuit is shown in the figures on the left.
- The experimental set-up for measuring the voltage and current in the parallel-tuned circuit is shown on the right.
- R_i denotes the internal resistance of the digital function generator, which is given in the operational instructions as $R_i = 2\Omega$.





Setup (3/3)

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For the digital function generator select following settings:

- DC-offset: $+/-0V$
- Amplitude $(U_{SS}) : 10V$
- Frequency: $0-10kHz$
- Mode: sinusoidal

The settings can be altered according to the experimenter's discretion. But is it important to leave the settings constant during the experiment. Especially the measuring ranges of the multimeter must remain the same during the measurement, because different ranges use different internal resistors! It is recommended to adjust the measuring range of the multimeter at the maximum of the resonance point (current for series tuned circuit and voltage for parallel tuned circuit) and leave them unchanged during the measurement.

Select the following measuring ranges on the Multimeter:

series-tuned circuit:

- Voltage (~): $3V$
- Current (~): $30mA$

parallel-tuned circuit:

- Voltage (~): $1V$
- Current (~): $10mA$

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Prodedure

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The voltage U and current I are measured according to the set-up for different frequencies f, which can be set and directly read off at the digital function generator. The frequency steps should get smaller when approaching the resonance frequency. Nevertheless it is recommended to determine the resonance frequencies for the different values of electrical components first, in order to have an idea how to choose the steps. For this one should use the quantity, which reaches its minimum at the resonance frequency. It is recommended to note all measurements for one quantity (e.g. the voltage) for the different frequencies first and then measure the other quantity (e.g. current) for the same frequencies.

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Results and evaluation

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Results

The resonance frequencies f_{res} are measured as follows:

- $0,1 \mu F$
  - series: $3401 Hz$
  - parallel: $3400 Hz$
- $0,47 \mu F$
  - series: $1552 Hz$
  - parallel: $1554 Hz$

Now the voltage drops U over the LC-components and the currents I through the circuits are
measured at different frequencies for the different set-ups:

- For the series-tuned circuit we obtain the following results:
- For the parallel-tuned circuit we obtain the following results:

Results

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Evaluation (1/9)

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Task 1: Determine the resonance frequency for both combinations of coil and capacitor and compare with the theoretical values:

Because the resonance frequency is the same for both series- and parallel-tuned circuits and is additionally independent of the resistors, one has to consider only two cases for the two capacitors. One gets:

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}$$

which in this case leads to the theoretical values $(L = 24mH)$:

- $C = 0,1\mu F : f_{res} = 3249Hz$
- $C = 0,47\mu F : f_{res} = 1499Hz$

Evaluation (2/9)

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Comparing the measured and the theoretical values, one finds that the results are within 5% deviation.
The plotted resonance curves are shown in Fig. 1 – Fig. 2*:*

Fig. 1a: Resonance curve of the current in the series tuned circuit with $C = 0.1\mu F$ (the values $I_{res}/\sqrt{2}$ for the bandwidth are plotted as $I = const.$ graphs)

Fig. 1b: Resonance curve of the current in the series tuned circuit with $C = 0.47\mu F$ (the values $I_{res}/\sqrt{2}$ for the bandwidth are plotted as $I = const.$ graphs)

Fig. 2a: Resonance curve of the voltage in the parallel tuned circuit with $C = 0.1\mu F$ (the values $U_{res}/\sqrt{2}$ for the bandwidth are plotted as $y = const.$ Graphs)

Fig. 2b: Resonance curve of the voltage in the parallel tuned circuit with $C = 0.47\mu F$ (the values $U_res/\sqrt{2}$ for the bandwidth are plotted as $y = const.$ Graphs)

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Evaluation (3/9)

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Task 2: Determine the impedance Z of the LC-component for both circuits with the measurements from task 1 and compare with the theoretical values.

The measured values Z_M of the impedance are simply derived by: $Z_M = \frac{U(f)}{I(f)}$

where $U(f)$ and $I(f)$ are the voltage and currrent measured in task 1 at the frequency f.

The theoretical value for the series tuned circuit is given by: $Z_S = |\omega L - \frac{1}{wC}|$

Evaluation (4/9)

PHYWE

One obtains for $C = 0,1\mu F$:

And for $C = 0,47\mu F$:

The plotted curves for the impendances for the series-tuned circuit are plotted in Fig. 3 -4.

Fig. 3: theoretical measured impedances in the series tuned circuits with $C = 0,1\mu F$

Fig. 4a and 4b: theoretical and measured impedances in the series tuned circuits with $C = 0.47\mu F$ (different scaling on the y-axis)

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Evaluation (5/9)

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In the parallel-tuned circuit we obtain the theoretical values for the impedance of the LC-component by using equ.

$$\frac{1}{Z_P} = |\frac{1}{iwL + iwC}|$$

We get:

The plotted curves for the impendances for the parallel-tuned circuit are plotted in Fig. 5a and 5b:

Fig. 5a: theoretical and measured impedances in the parallel-tuned circuits with $C = 0.1\mu F$

Fig. 5b: theoretical and measured impedances in the parallel-tuned circuits with $C = 0.47\mu F$

Evaluation (6/9)

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Summary:

One can see, that the values of the measured impedances partially differ widely from the theoretical values, nevertheless the general behaviour is confirmed. The biggest deviations are present near the resonance frequency, where the ohmic part of the impedance (the ohmic resistance of the coil) contributes more than at the edges of the plotted curves.

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Evaluation (7/9)

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Task 3: Determine the bandwidth B and Q-factor for the series-tuned circuit from the resonance curve and compare with the theoretical values

The theoretical values for the quality factor Q are given by equ. $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$, but before inserting the values, one must consider the different parts which contribute to the total resistance. These are the ohmic resistor R itself, the real part of the impedanze at the resonance point, here simply denoted as R_{LC} , which is simply given by $R_{LC} = \frac{U_{res}}{I_{res}}$ and the internal resistance of the function generator R_i. Therefore

$$Q = \frac{1}{R_{tot}}\sqrt{\frac{L}{C}}$$

with $R_{tot} = R + R_i + R_{LC} = R + R_i + \frac{U_{res}}{I_{res}}$

Evaluation (8/9)

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The measured value of the quality factor Q_M is calculated with equ. $Q = \frac{F_{res}}{B}$. The frequencies f_1 and f_2 for the bandwith are determined from the plot in Fig. 1 & 2.

For the series-tuned case with $C = 0,1\mu F$ one gets:

The theoretical and measured values coincide quite well (deviation within 5% in the circuit with $R = 47\Omega$ and within 7% in the circuit with $R = 100\Omega$).

For the series-tuned case with $C = 0.47\mu F$ one gets:

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