Balmer series / Determination of Rydberg's constant

(Item No.: P2510700)

Curricular Relevance

Area of Expertise:	Education Level:	Topic:	Subtopic:	Experiment:
Physics	University	Modern Physics	Atomic Physics	Balmer series / Determination of Rydberg's constant

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	1 Hour	2 Hours	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Diffraction image of a diffraction grating, visible spectral range, single electron atom, atomic model according to Bohr, Lyman-, Paschen-, Brackett- and Pfund-Series, energy level, Planck’s constant, binding energy

Overview

Short description

Principle

The spectral lines of hydrogen and mercury are examined by means of a diffraction grating. The known spectral lines of mercury are used to determine the grating constant. The wave lengths of the visible lines of the Balmer series of hydrogen are measured.

Fig. 1: Set-up of experiment P2510700

Equipment

Position No.	Material	Order No.	Quantity
1	PHYWE High voltage supply unit with digital display DC: 0... ± 10 kV, 2 mA	13673-93	1
2	Object holder, 5x5 cm	08041-00	1
3	Spectrum tube, hydrogen	06665-00	1
4	Spectrum tube, mercury	06664-00	1
5	Diffraction grating, 600 lines/mm	08546-00	1
6	Tripod base PHYWE	02002-55	1
7	Insulating support	06020-00	2
8	Barrel base PHYWE	02006-55	1
9	Cover tube for spectral tubes	06675-00	1
10	Stand tube	02060-00	1
11	Meter scale, demo. l=1000mm	03001-00	1
12	Holders for spectral tubes,1 pair	06674-00	1
13	Connecting cord, 30 kV, 1000 mm	07367-00	2
14	Support rod PHYWE,square,l 400mm	02026-55	1
15	Right angle clamp PHYWE	02040-55	3
16	Cursors, 1 pair	02201-00	1
17	Measuring tape, l = 2 m	09936-00	1

Tasks

1. Determine the diffraction grating constant by means of the mercury spectrum.
2. Determine the visible lines of the Balmer series in the hydrogen spectrum, of Rydberg’s constant and of the energy levels.

Setup and Procedure

The experimental set-up is shown in Fig. 1. Hydrogen or mercury spectral tubes connected to the high voltage power supply unit are used as a source of radiation. The power supply is adjusted to about 5 kV. The scale is attached directly behind the spectral tube in order to minimize parallax errors. The diffraction grating should be set up at about 50 cm and at the same height as the spectral tube. The grating must be aligned so as to be parallel to the scale. The luminous capillary tube is observed through the grating (see Fig 2).

The room is darkened to the point where it is still possible to read the scale. The distance $2I$ between spectral lines of the same color in the right and left first order spectra are read through the grating. The distance between the grating and the eye should be so short, that both lines are visible at the same time **without moving the head.** The distance d between the scale and the grating is also measured.

Three lines are clearly visible in the Hg spectrum. The grating constant g is determined by means of the wavelengths given in Table 1. Rydberg’s constant, and thus the energy levels in hydrogen, are determined from the measured wavelengths by means of Balmer’s formula.

Tab. 1: Determination of the grating constant from the wavelengths of the Hg spectrum

Color	λ/nm	$2I/mm$	$g/\mu m$
yellow	578.0	330	1.680
green	546.1	311	1.672
blue	434.8	244	1.661

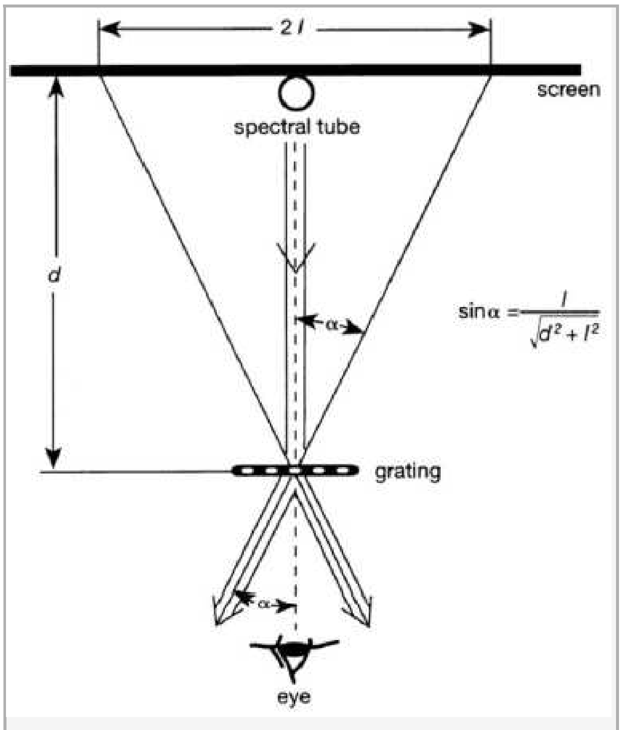

Fig. 2: Diffraction at the grating.

Theory and evaluation

1. Diffraction grating

If light of wavelength I impinges on a grating with constant g , it is diffracted. Intensity peaks occur when the angle of diffraction α fulfills the following condition:

$n \cdot \lambda = g \cdot \sin\alpha$; $n = 0, 1, 2, \ldots$ (1)

Light is collected by the eye on the retina, therefore the light source is seen in the color of the observed spectral line on the scale in the prolongation of the light beams. For the diffraction of the nth order, the following relation is deduced from the geometrical structure (Fig. 2):

$n \cdot \lambda = g \cdot (\frac{l}{\sqrt{d^2+l^2}})$ (2)

In the examples given in Table 1, the average obtained for the three measurements of the grating constant is $g = 1.671 mm$.

2. Hydrogen spectrum

Due to collision ionization, H_2 is converted to atomic hydrogen in the spectral tube. Electrons from the H atoms are excited to higher energy levels through collisions with electrons. When they return to lower energy levels, the atoms emit light of frequency f given by the energy difference of the concerned states:

$E = h \cdot f$ (3)

where h is Planck’s constant.

Applying Bohr’s atomic model, the energy E_n of a permitted electron orbit is given by:

$E_n = \frac{1}{8}\frac{e^4 m_e}{\varepsilon_0^2 h^2 n^2}$, $n = 1, 2, 3 \ldots$ (4)

where $\varepsilon_0 = 8.8542 \cdot 10^{-34} As/Vm$ is the electric field constant, $e = 1.6021 \cdot 10^{-19} C$ is the electronic charge and $m_e = 9.1091 \cdot 10^{-31} \mathrm{kg}$ is the mass of the electron at rest. The emitted light can therefore have the following frequencies:

$f_{\mathrm{nm}} = \frac{1}{8}\frac{e^4 m_e}{\varepsilon_0^2 h^3}(\frac{1}{n^2} - \frac{1}{m^2})$, $n, m = 1, 2, 3 \ldots$ (5)

If the wave number $N = I - 1$ is used instead of the frequency f , substituting $c = I \cdot f$ one obtains:

$N = R_{\mathrm{th}}(\frac{1}{n^2} - \frac{1}{m^2})$ where $R_{\mathrm{th}} = \frac{1}{8}\frac{e^4 m_e}{\varepsilon_0^2 h^3 c} = 1.097 \cdot 10^7 m^{-1}$ (6)

Here R_{th} is Rydberg's constant, which follows from Bohr's atomic model.

:Lyman series

Spectral range: ultraviolet

$n = 2$:Balmer series

Spectral range: ultraviolet till red

$n = 3$:Paschen series

Spectral range: infrared

$n = 4$:Bracket series

Spectral range: infrared

$n = 5$:Pfund series

Spectral range: infrared

Fig. 3 shows the energy level diagram and the spectral series of the H atom. For $m \to \infty$, one obtains the limits of the series; the associated energy is thus the ionization energy (or the binding energy) for an electron in the nth permitted orbit. The binding energy can be calculated by means of the equation:

$$E_n = -R_{\mathrm{th}} \cdot h \cdot c \frac{1}{n^2}$$

where $c = 2.99795 \cdot 10^8 m/s$ and $h = 6.6256 \cdot 10^{-34} Js = 4.13567 \cdot 10^{-15} \mathrm{eV}$ s. The ground state is found to be 13.6 eV.

Tab. 2: Examples of measurements for the H spectrum

(Balmer series) Distance d = 450 mm

Line	$2I$	λ_{exp}	λ_{lit}	R_{exp}
H_α	384 mm	656 nm	656.28 nm	
H_β	275 mm	489 nm	486.13 nm	$1.093 \cdot 10^7 m^{-1}$
H_γ	243 mm	436 nm	434.05 nm	$1.092 \cdot 10^7 m^{-1}$
H_σ	-	-	410.17 nm	-

average: Rexp = 1.094 · 107 m^{-1}

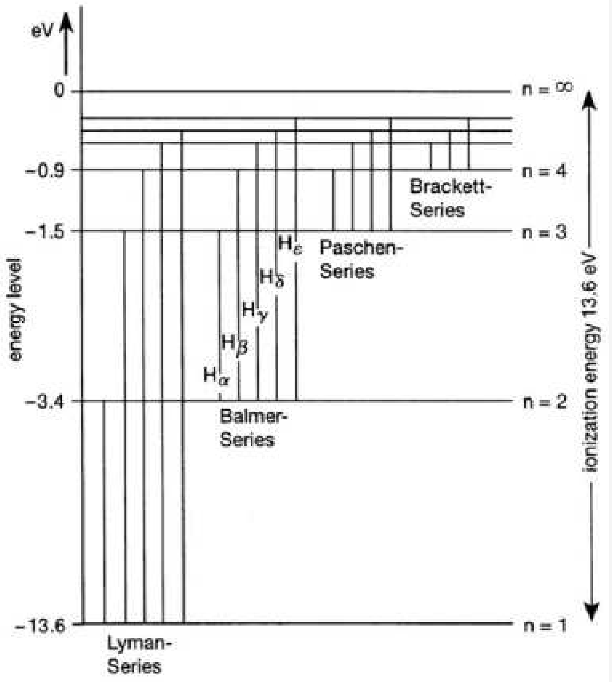

Fig. 3: Energy level diagram of the H atom.

Note

- If the room is sufficiently darkened, next to the atomic hydrogen spectrum, the molecular H2 band spectrum may be observed. The numerous lines, which are very close to each other, are due to the oscillations of the molecule.
- The H_σ line is situated on the border of the visible spectral range and is too weak to be ob-served by simple methods.
- The treatment of more complex atoms requires quantum mechanics. In this case, the energies of the states are determined by the eigenvalues of the hamiltonian of the atom. For atoms similar to hydrogen, calculations yield the same results as Bohr’s atomic model.